Fat Magnon

Shinji Hirano

Niels Bohr Institute, Copenhagen

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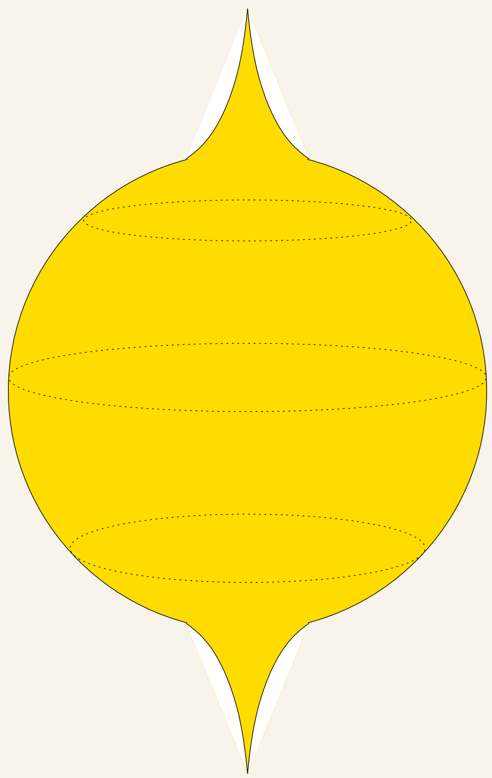
Fat magnon in a nutshell



¶ A D-brane type state which shares the characteristic of the giant magnon of Hofman and Maldacena

¶ More specifically, it is a bound state of giant graviton (spherical D3-brane) and giant magnon (open F-string) – (topologically) spherical D3-brane with electric flux

¶ Its anomalous dimension is exactly the same as that of the giant magnon

Introduction

¶ The giant magnon is the string theory dual of magnon in the *infinite* chain at *large* 't Hooft coupling

- Why important?

¶ It goes beyond the BMN/pp-wave and (Frolov-Tseytlin) fast spinning string limits

◇ In these limits, $\lambda' = \lambda/J^2$ fixed finite while λ, J taken to ∞, and the semi-classical $(\alpha' \sim 1/\sqrt{\lambda} \ll 1)$ string energy E turns out to admit the λ' expansion

$\Longrightarrow$ The comparison between string theory and gauge theory in the *weak* coupling expansion is possible

$\Longrightarrow$ It, however, limits us to consider only *low* momentum magnons

◇ Magnons constitute the essential basis for the Bethe ansatz. So in order to understand the integrability of the full string theory on $AdS_5 \times S^5$, it would be necessary to go beyond these limits and understand the magnons at arbitrary momenta in the string theory side

¶ We would then have to go beyond the weak coupling expansion also in the gauge theory side

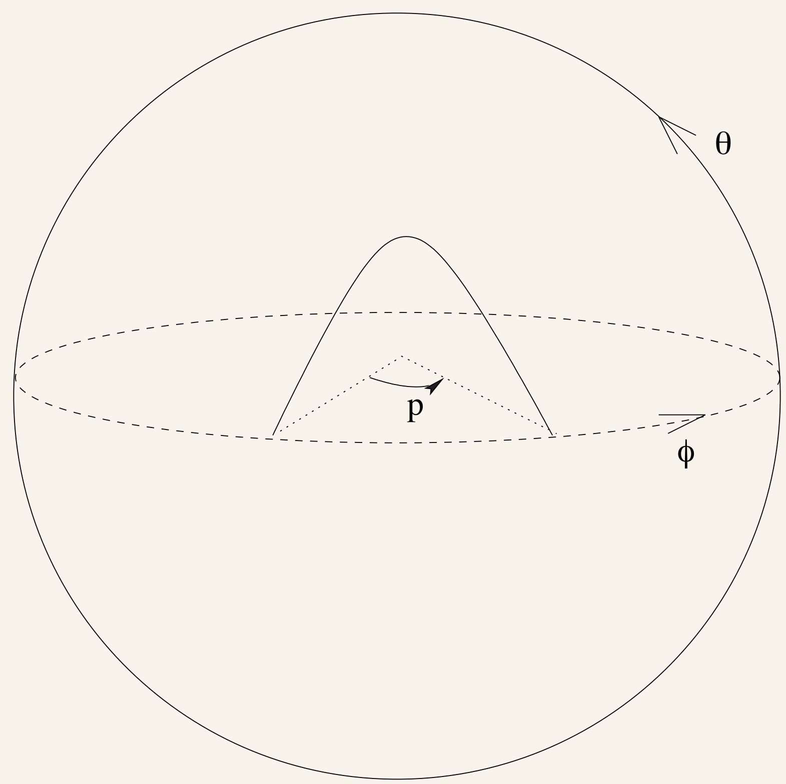
◇ At large 't Hooft coupling the spin chain is very long-ranged. The spin chain Hamiltonian/dilatation operator is practically incalculable, as it requires the all-loop SYM computation

◇ Nevertheless the all-loop (asymptotic) Bethe ansatz was guessed from the spin chain perspective guided by the integrability, BMN scaling, and a few loop order results in SYM (B(eisert)D(ippel)S(taudacher), BS)

◇ Remarkably the all-loop guess was later *derived* by Beisert only by the use of supersymmetry without need of knowing the detailed dynamics of $\mathcal{N} = 4$ SYM except for the inspiring inputs from it (cf. another intriguing development – Hubbard model)

$\Longrightarrow$ It is now possible to quantitatively compare string theory and gauge/spin chain results at *large* 't Hooft coupling

¶ Indeed Hofman and Maldacena found that the string theory dual of magnon in the limit, $N \longrightarrow \infty$ first and then $J \longrightarrow \infty$ with λ large but fixed finite – infinite chain and very long-ranged – is a macroscopic open string orbiting in S^5 (giant magnon), and the magnon momentum p is the geometric angle between two endpoints of the string



◇ Anomalous dimension/dispersion relation:

$$E - J = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p}{2}\right| \quad \xleftarrow{\lambda\gg 1} \quad \sqrt{1+\frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$$

◇ At first sight it might have appeared that the periodicity in momentum suggested the discrete worldsheet in which the lattice spacing is to set the period. But it does not. p is periodic, as p is canonical conjugate to the angular momentum J, thus a geometric angle

$\Longrightarrow$ We might then hope that further studies of the giant magnons will lead us to the better understanding of the integrability and Bethe ansatz for the $AdS_5 \times S^5$ string.

¶ Several works on the giant magnons have appeared:

(1) multi-magnon bound states/giant magnon with two or three angular momenta (Dorey ....)

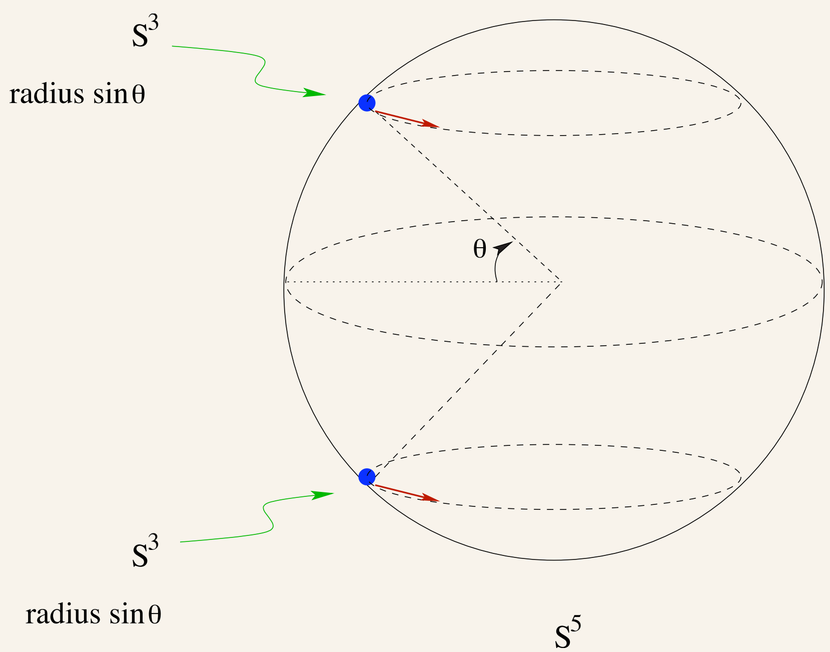
(2) stringy ($\alpha' = 1/\sqrt{\lambda}$) corrections – agreement (Minahan et al)

(3) finite chain – status not very clear (Arutyunov et al)

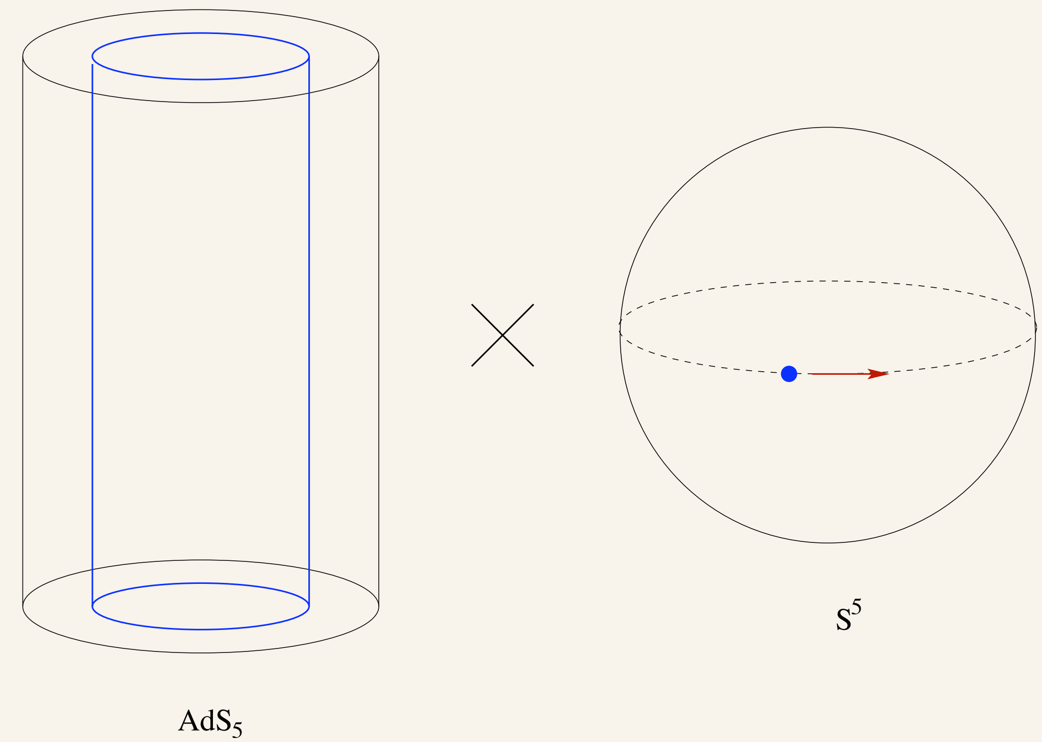
(4) $SL(2)$ sector, β-deformed background, M-theory, Melvin etc (......)

¶ The D-brane type states play import roles in AdS/CFT

◇ Giant graviton – spherical D3-brane orbiting in S^5 and expanded either in S^5 or AdS_5 (McGreevy, Susskind, Toumbas; Grisaru, Myers, Tafjord; Hashimoto, SH, Itzhaki)



Sphere Giant



AdS Giant

♮ These are states degenerate with a gravion mode in S^5:

Quantum numbers $E = J$. Graviton mode $\sim$ TrZ^J, Sphere giant $\sim$ Tr$_{A^J}Z$ ((sub-)determinant), AdS giant $\sim$ Tr$_{S^J}Z$

♮ size $= R_{AdS}\sqrt{\frac{J}{N}}$

♮ In the semi-classical approximation, the graviton mode is the more adequate description for the low energy $E \sim \mathcal{O}(1)$ and the giant gravitons for the high energy $E \sim \mathcal{O}(N)$

◇ "Giant" Wilson loop (circular or a straight line) – D3 and D5-brane with electric flux (Drukker, Fiol; Yamaguchi)

♮ The shape: $AdS_2 \times S^2$ for D3-brane and $AdS_2 \times S^4$ for D5-brane

♮ Wilson loop $\sim$ Tr U, D3 "giant" loop $\sim$ Tr U^k or Tr$_{S^k}U$, D5 "giant" loop $\sim$ Tr$_A U$ where $U = P\exp\left(\int_C ds(A_\mu \dot{x}^\mu + \Phi|\dot{x}|)\right)$

$\Longrightarrow$ It is then natural to ask what is the analogue of these D-brane type states for the giant magnon. Therefore ...

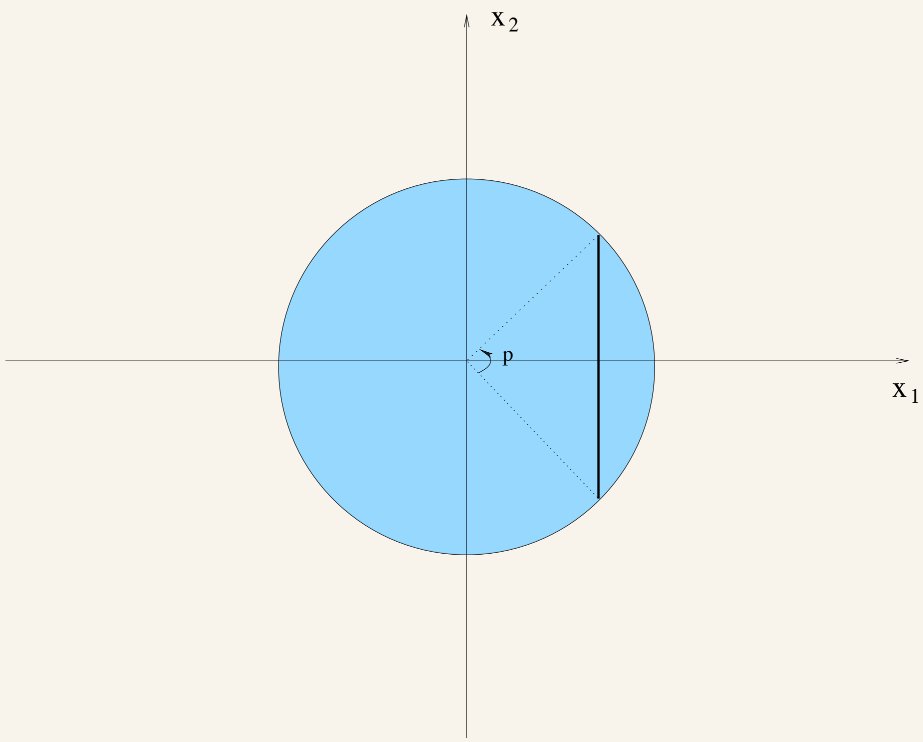
♠ In this talk I discuss a D-brane type state which is a giant version of the giant magnon (Fat magnon)

Giant magnon – review (Hofman, Maldacena)

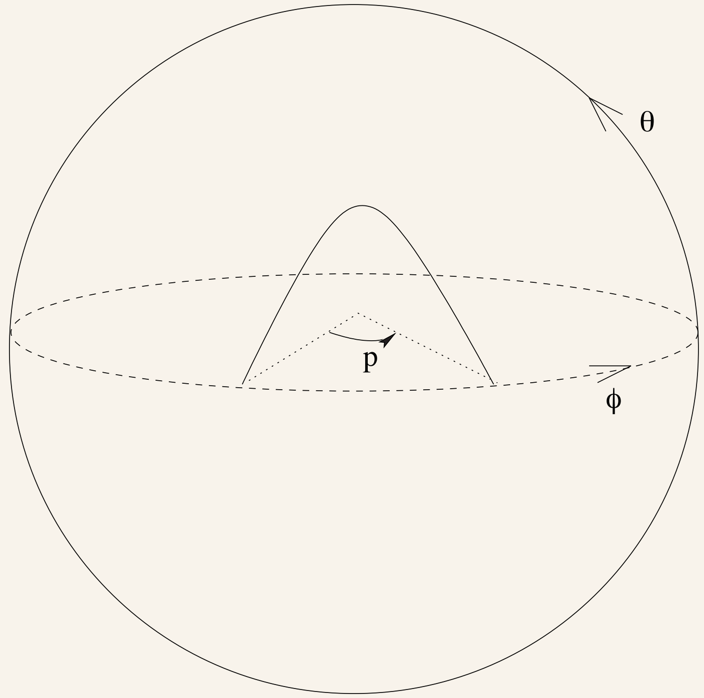
¶ We work in the probe approximation

¶ We found it particularly convenient to adopt LLM (Lin, Lunin, Maldacena – bubbling AdS) coordinates

- What we are going to find is ...



Giant magnon in LLM coordinates



In the standard coordinates

◇ The part of spacetime to embed the giant magnon (string) – $R_{time} \times S^5$ in LLM coordinates:

$$ds^2 = R^2\left[-\left(1-r^2\right)\left(dt - \frac{r^2}{1-r^2}d\tilde{\phi}\right)^2 + \frac{dr^2 + r^2 d\tilde{\phi}^2}{1-r^2} + \left(1-r^2\right)d\tilde{\Omega}_3^2\right]$$

♮ The transformations $r = \cos\theta$ and $\tilde{\phi} = \phi - t$ to go to the standard spherical coordinates

◇ The ansatz for the shape and dynamics of the string

$$r = r(\sigma)\ , \qquad \phi = \phi(\tau, \sigma)$$

$\Longrightarrow$ The Nambu-Goto action (in the static gauge $t = \tau$)

$$S_{NG} = -\frac{\sqrt{\lambda}}{2\pi}\int d\tau d\sigma \sqrt{\frac{r'^2}{1-r^2} + r^2\phi'^2 - \frac{r'^2 r^2}{1-r^2}\dot{\phi}^2}$$

◇ The giant magnon solution (boundary condition – two endpoints sit on the edge of the droplet/equator of S^5):

$$x_2(\sigma) = \frac{2}{\pi}\sqrt{1-x_1^2}\left(\sigma - \frac{\pi}{2}\right)\ , \qquad x_1(\sigma) = \text{const}$$

modulo rotation in the (x_1, x_2) plane where $(x_1, x_2) = (r\cos\tilde{\phi}, r\sin\tilde{\phi})$

◇ Divergent angular momentum and energy:

$$\pi_\phi = \frac{\sqrt{\lambda}}{2\pi}\frac{x_2^2 x_2'}{1-r^2}\ , \qquad \mathcal{H} = \pi_\phi - \mathcal{L}$$

$\Longrightarrow J = \int d\sigma \pi_\phi$ and $E = \int d\sigma \mathcal{H}$ divergent due to the contribution from the endpoints of the string

♮ $J = \infty \Longleftrightarrow$ infinite chain: This is the limit we are interested in

◇ The anomalous dimension:

$$E - J = -\int d\sigma \mathcal{L} = \frac{\sqrt{\lambda}}{\pi}\sqrt{1-x_1^2} = \frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2}$$

¶ Note: The probe approximation is not really valid near the endpoints of the string – the energy density is large

Fat magnon

∗ We wish to find a D-brane type state which shares the characteristic of the giant magnon

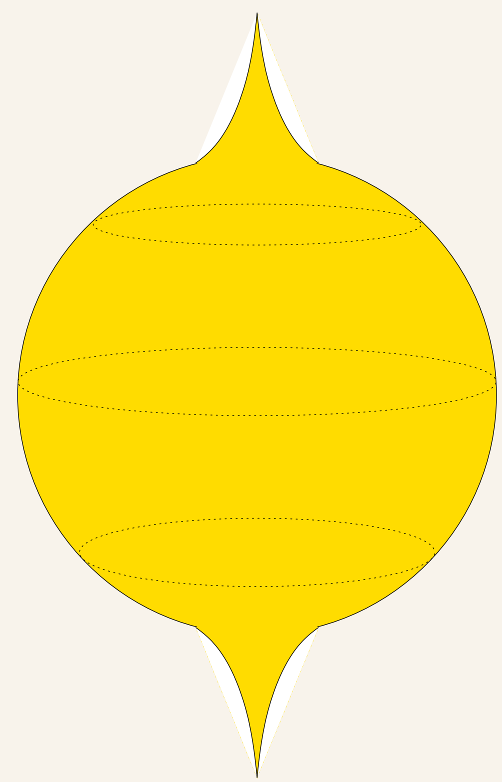
∗ A natural D-brane to consider is a giant graviton (spherical D3-brane), since it carries the angular momentum in S^5 which together with the energy are the only quantum numbers it has

∗ To be a magnon-like state, it is essential to have the characteristic geometric angle in S^5 for the object of our concern, which corresponds to the magnon momentum p

∗ We achieve that by starting from the sphere giant – spherical D3-brane and turning on the electric flux – attaching F-strings – which elongates the spherical shape to develop an open angle

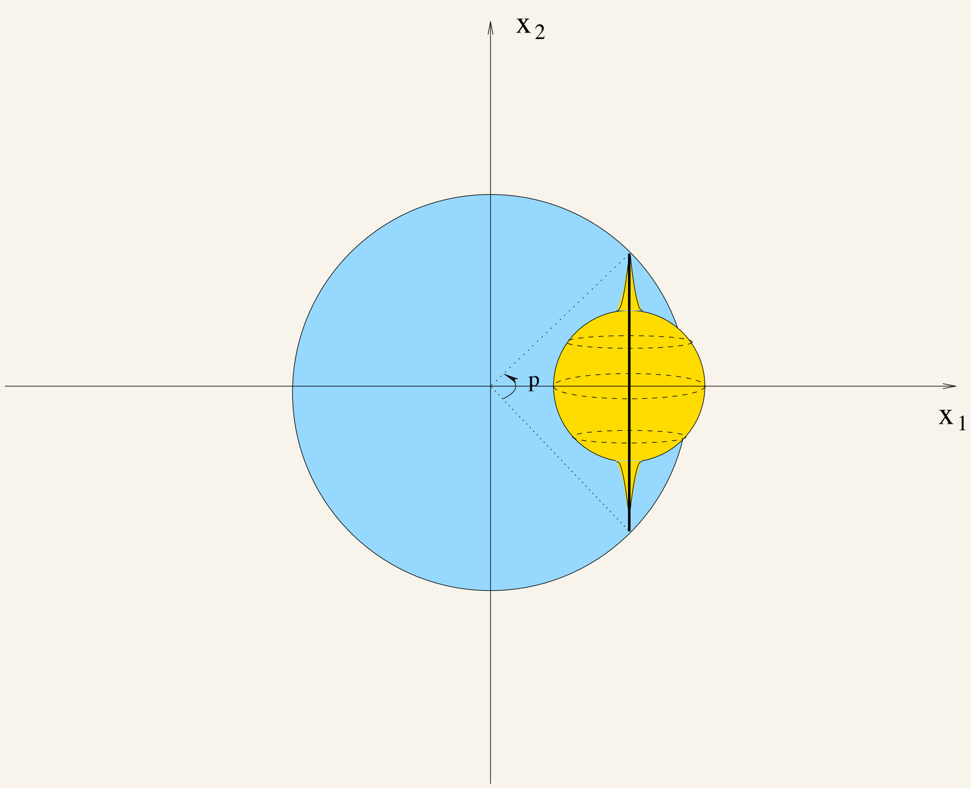
∗ So the object is going to be a bound state of sphere giant and giant magnon, as we will see more in detail

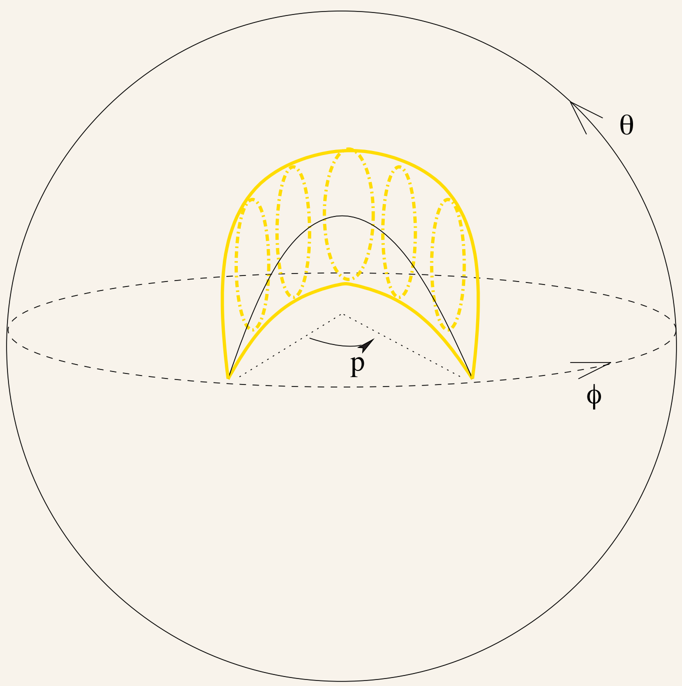
∗ It is indeed a variant of BIon (Callan, Maldacena). Two endpoints of the giant magnon bound to the giant graviton corresponds to a pair of unlike electric charges put at the antipodal points in S^3. They will develop the spikes as in the BIon case. So the fat magnon will look like ...



¶ We again work in the probe approximation

- What we are going to find this time is ...





Fat magnon in LLM coordinates

In the standard coordinates

◇ Again the part of spacetime to embed the fat magnon (topologically spherical D3-brane) – $R_{time} \times S^5$ in LLM coordinates:

$$ds^2 = R^2 \left[-\left(1-r^2\right)\left(dt - \frac{r^2}{1-r^2} d\widetilde{\phi}\right)^2 + \frac{dr^2 + r^2 d\widetilde{\phi}^2}{1-r^2} + \left(1-r^2\right)\left(d\chi^2 + \sin^2\chi d\widetilde{\Omega}_2^2\right) \right]$$

◇ The worldvolume coordinates $(\tau, \sigma_1, \sigma_2, \sigma_3)$ and the static gauge

$$t = \tau, \quad \chi = \sigma_1 \equiv \sigma \ , \quad \widetilde{\Omega}_2 = (\sigma_2, \sigma_3)$$

◇ The ansatz for the shape and dynamics of the D3-brane

$$r = r(\sigma) \ , \qquad \phi = \phi(\tau, \sigma)$$

♮ This ansatz assumes the $SO(3)$ symmetry of S^2

$\Longrightarrow$ The D3-brane action (DBI+CS)

$$\begin{aligned} S_{D3} &= -T_3 \left[\int d\tau d^3\sigma \sqrt{-\det(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\pi l_s^2 F_{ab})} + \int C_4 \right] \\ &= -\frac{2}{\pi} N \int d\tau d\sigma \sin^2\sigma \left[(1-r^2)\sqrt{\mathcal{D}} - (1-r^2)^2 \dot{\phi} \right] \end{aligned}$$

where

$$\mathcal{D} = \frac{r'^2}{1-r^2} + r^2\phi'^2 - \frac{r'^2}{1-r^2} r^2 \dot{\phi}^2 - \left(\frac{2\pi}{\sqrt{\lambda}}\right)^2 F_{\tau\sigma}^2 + (1-r^2)(1-r^2\dot{\phi}^2)$$

◇ The fat magnon solution

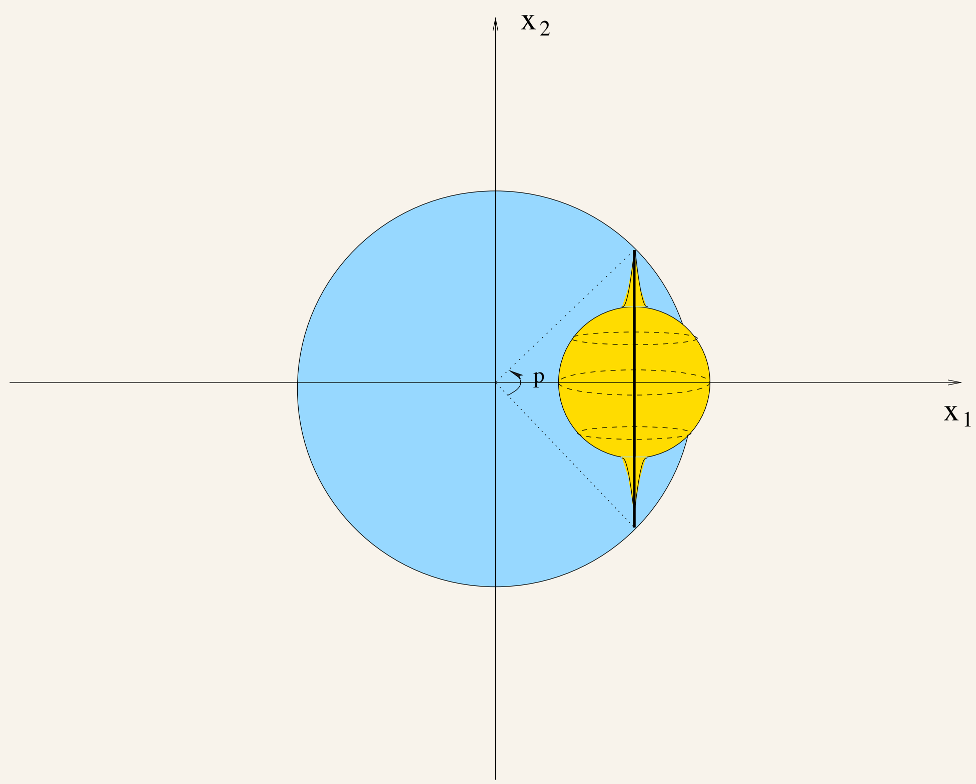
$$x_2(\sigma) = c - \kappa \cot\sigma \ , \quad x_1(\sigma) = \text{const} \ , \quad \frac{2\pi}{\sqrt{\lambda}} F_{\tau\sigma} = x_2'$$

modulo rotation in the (x_1, x_2) plane where $(x_1, x_2) = (r\cos\widetilde{\phi}, r\sin\widetilde{\phi})$, as before

◇ The boundary condition (only considering the case $c = 0$)

$$\sigma_0 \leq \sigma \leq \pi - \sigma_0 \ , \qquad x_2(\sigma_0) = -\sqrt{1 - x_1^2} \quad \Big(\text{and so} \quad x_2(\pi - \sigma_0) = \sqrt{1 - x_1^2}\Big)$$

♮ The spikes end at the edge of the droplet/equator of S^5



◇ The flux quantization (k the number of F-strings)

$$\pi_A \equiv \frac{\partial \mathcal{L}}{\partial \dot{A}_\sigma} = \frac{4N}{\sqrt{\lambda}}\kappa = k \quad \Longrightarrow \quad \kappa = \frac{\sqrt{\lambda}}{4N}k$$

◇ Divergent angular momentum and energy

$$\pi_\phi = T \sin^2\sigma \left[\frac{r^2 r'^2}{1-r^2} + (1-r^2)\right] , \qquad \mathcal{H} = kF_{\tau\sigma} + \pi_\phi$$

$\Longrightarrow J = \int d\sigma \pi_\phi$ and $E = \int d\sigma \mathcal{H}$ divergent due to the contribution from the endpoints of the spikes

◇ The anomalous dimension

$$E - J = \int d\sigma F_{\tau\sigma}\pi_A = k\frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2}$$

♮ For $k = 1$, exactly the same as the giant magnon case

The bound state interpretation

¶ We will now manifestly see how the giant graviton and giant magnon are bound. What we will show is

$$J_{fm} = J_{gg} + J_{gm} \ , \qquad E_{fm} = E_{gg} + E_{gm}$$

that is, the fat magnon is a threshold (BPS) bound state

¶ A closer look at the angular momentum

$$\pi_\phi = T \sin^2\sigma \left[\frac{r^2 r'^2}{1-r^2} + (1-r^2) \right] = k \frac{\sqrt{\lambda}}{2\pi} \frac{x_2^2 x_2'^2}{1-r^2} + T \sin^2\sigma \, (1-r^2)$$

$\Longrightarrow$ The first term $= k \times$ (π_ϕ^{gm} of giant magnon)

$\Longrightarrow$ The second term to be identified with the giant graviton contribution

◇ Recall that we are in the strict $N \to \infty$ limit

$\Longrightarrow \kappa = \frac{\sqrt{\lambda}}{4N} k \to 0$ provided that $k \sim \mathcal{O}(1)$. To comply with the boundary condition, the limit should be taken as

$$\kappa \, , \sigma_0 \to 0 \qquad \text{keeping} \quad \kappa \cot \sigma_0 = \sin \frac{p}{2} \quad \text{fixed}$$

$\Longrightarrow$ The second term of the fat magnon momentum becomes

$$q \equiv \int d\sigma T \sin^2 \sigma \, (1 - r^2) = N \sin^2 \frac{p}{2}$$

• Why is this the contribution from the giant graviton (other than the what-else logic)?

◇ The size/angular momentum relation of giants (McGreevy, Susskind, Toumbas)

$$R_{gg} = R_{AdS} \times \sqrt{\frac{q}{N}}$$

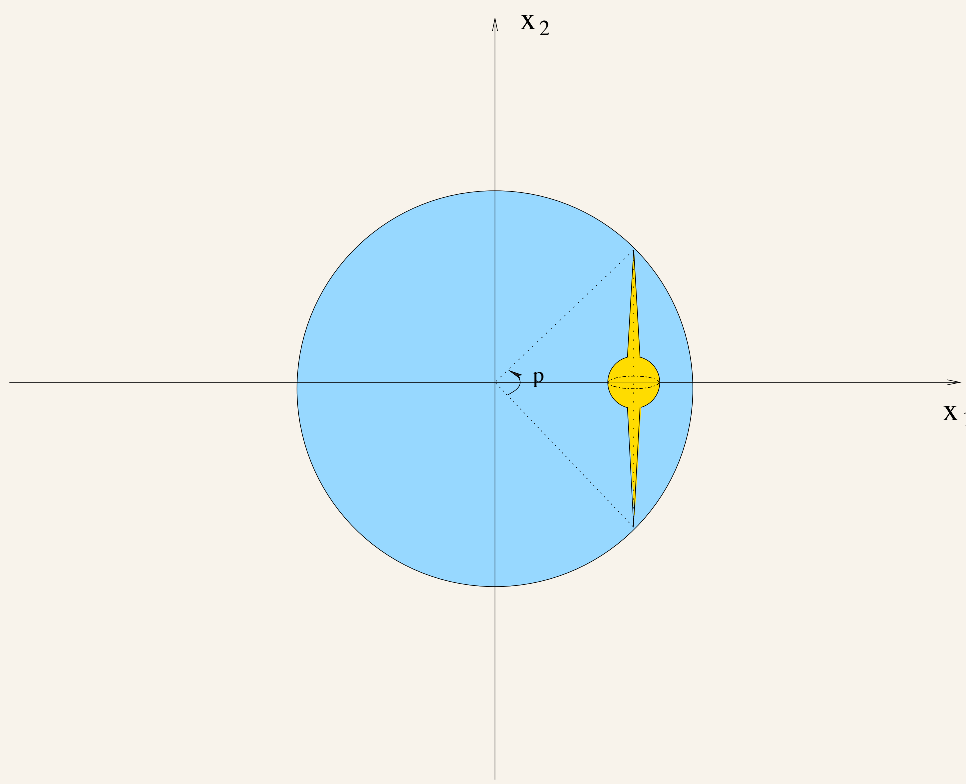
◇ while the (varying) size of the elongated S^3 of the fat magnon

$$R_{fm} = R_{AdS} \times \sqrt{1 - x_1^2 - x_2(\sigma)^2} \, , \qquad \text{where} \qquad x_2(\sigma) = -\kappa \cot \sigma$$

$\Longrightarrow$ In the limit we are taking, $x_2(\sigma)$ is zero except at the north and south poles. So the fat magnon is nearly a perfect S^3 emenating sharp spikes at the poles. Except at the poles indeed

$$R_{fm|bulk} = R_{AdS} \sin\frac{p}{2} = R_{AdS} \times \sqrt{\frac{q}{N}}$$

in accordance with the giant graviton size/angular momentum relation



Fat magnon in LLM coordinates near the limit

$\Longrightarrow$ In sum, we found that

$$J_{fm} = J_{gm} + J_{gg} \qquad \text{and} \qquad E_{fm} = \Delta_{anom} + J_{gm} + J_{gg}$$

$\Longrightarrow$ Since $E_{gg} = J_{gg}$ and $E_{gm} = \Delta_{anom} + J_{gm}$, as we promised

$$E_{fm} = E_{gm} + E_{gg}$$

that is, the fat magnon is a threshold (BPS) bound state

¶ Note: The probe approximation is not really valid near the spikes – the energy density and curvature are large. But it is just as bad as the giant magnon case

The dual CFT operator

¶ We wish to find a conceivable proposal for the dual CFT operator for the fat magnon

◇ Three ingredients in the fat magnon:

(1) giant magnon

(2) sphere giant (giant gravion in S^5)

(3) attaching the giant magnon (open string) to the giant graviton (D3-brane)

$\Longrightarrow$ The logical step to take is to understand the dual CFT operators for (1) the giant magnon, (2) sphere giant, and (3) sphere giant with open strings attached, and combine them together. Each one of them is known

(1) Giant magnon: Bethe ansatz form (Minahan, Zarembo)

$$\mathcal{O}_p = \sum_l e^{ipl} (\cdots ZZWZZ \cdots)$$

where l denotes the site of the spin excitation W. Note: no trace

(2) Giant graviton: (sub-)determinant = trace over antisymmetric representation (dim = angular momentum q) (Balasubramanian, Berkooz, Naqvi,

Strassler; Corley, Jevicki, Ramgoolam; Berenstein)

$$\mathcal{O}^{gg} = \epsilon^{i_1 i_2 \cdots i_{q-1} i_q}_{j_1 j_2 \cdots j_{q-1} j_q} Z_{i_1}^{\ j_1} Z_{i_2}^{\ j_2} \cdots Z_{i_q}^{\ j_q}$$

where $\epsilon^{i_1 i_2 \cdots i_{q-1} i_q}_{j_1 j_2 \cdots j_{q-1} j_q} = \Sigma_{\sigma \in S_q} (-1)^\sigma \delta^{i_1}_{\sigma(j_1)} \delta^{i_2}_{\sigma(j_2)} \cdots \delta^{i_{q-1}}_{\sigma(j_{q-1})} \delta^{i_q}_{\sigma(j_q)}$

♮ sphere giant = a hole in the free fermion system coming from the matrix (Z) quantum mechanics; eigenstates = $\chi_R(Z)$, hole $\longleftrightarrow$ antisymmetric representation R (CJR; B; LLM; Takayama, Tsuchiya)

(3) Sphere giant with an open string excitation on it (Balasubramanian, Huang, Levi, Naqvi + Berenstein, Feng)

$$\mathcal{O}^{gg+open} = \epsilon^{i_1 i_2 \cdots i_{q-1} i_q}_{j_1 j_2 \cdots j_{q-1} j_q} Z_{i_1}^{\ j_1} Z_{i_2}^{\ j_2} \cdots Z_{i_{q-1}}^{\ j_{q-1}} \mathcal{O}[\Phi_i]_{i_q}^{\ j_q}$$

$\mathcal{O}[\Phi_i]$ is a monomial ("word") composed of the real adjoint scalars $\Phi_{i=1,\cdots,6}$, corresponding to the open string excitation in S^5

♮ This is based on the fact that the frequencies of the fluctuation spectrum is independent of the size of giants: $\omega_k \sim k/R$ (k = angular momentum in S^3). (Das, Jevicki, Mathur)

◇ Having understood these, the most naive guess would then be

$$\mathcal{O}_p^{fat} \stackrel{?}{=} \lim_{\substack{N\to\infty, q\to\infty \\ q/N=\sin^2\frac{p}{2}}} \epsilon^{i_1 i_2\cdots i_q i_{q+1}}_{j_1 j_2\cdots j_q j_{q+1}} Z_{i_1}^{\ j_1} Z_{i_2}^{\ j_2}\cdots Z_{i_q}^{\ j_q} (\mathcal{O}_p)_{i_{q+1}}^{\ j_{q+1}}$$

$\Longrightarrow$ This fails. At least two things are wrong:

(1) rewritten in term of multi-traces, implying that the phase e^{ipl} is trivial

(2) Z at the beginning and/or end of the word, yielding the breakup of $\mathcal{O}_p^{fat}$ into a sphere giant ($\epsilon Z\cdots Z$) with a closed string emission ($\text{Tr}\mathcal{O}'_p$) and a larger giant with an open string excitation on it ($\epsilon Z\cdots Z\mathcal{O}'_p$)

$$\epsilon Z\cdots Z(\mathcal{O}'_p Z) \sim (\epsilon Z\cdots ZZ)\text{Tr}\mathcal{O}'_p + \epsilon Z\cdots ZZ\mathcal{O}'_p$$

$\Longrightarrow$ No p dependence in the anomalous dimension

- What's the resolution?

◇ Could it be

$$\mathcal{O}_p^{fat} \stackrel{?}{=} \lim_{\substack{N\to\infty, q\to\infty \\ q/N=\sin^2\frac{p}{2}}} \epsilon^{i_1 i_2\cdots i_q i_{q+1}}_{j_1 j_2\cdots j_q j_{q+1}} Z_{i_1}^{\ j_1} Z_{i_2}^{\ j_2}\cdots Z_{i_q}^{\ j_q} (\bar{Z}\mathcal{O}_p\bar{Z})_{i_{q+1}}^{\ j_{q+1}}$$

This may avoid both of the problems, but how can I convincingly argue for the insertion of $\bar{Z}$s. Doesn't look like BPS ...

or the operator like

$$\mathcal{O}_p^{fat} \stackrel{?}{=} \lim_{\substack{N\to\infty, q\to\infty \\ q/N=\sin^2\frac{p}{2}}} \epsilon^{i_1 i_2 \cdots i_q i_{q+1} i_{q+2}}_{j_1 j_2 \cdots j_q j_{q+1} j_{q+2}} Z_{i_1}{}^{j_1} Z_{i_2}{}^{j_2} \cdots Z_{i_q}{}^{j_q} \left(\mathcal{O}_p\right)_{i_{q+1}}{}^{j_{q+1}}$$

Remember that the gauge theory operator dual to the giant magnon is not gauge-invariant (not traced). This may avoid the first problem, but there still seems to remain a similar problem as (2)

No conclusion for the dual CFT operator yet

Summary

¶ We found a new D-brane type state in AdS/CFT/spin chain triality, which is a giant version of the giant magnon

¶ It is shown to be a threshold BPS bound state of the sphere giant and giant magnon

¶ The scattering of fat magnons remains to be understood

¶ It is likely to be able to find the fat generalization of the bound state of multi-magnons (two or three angular momenta)

¶ It would be nice if we can understand the fat magnon in terms of spin chain (A natural place to start with would be Berenstein's proposal of the length varying spin chain)

¶ Any relation to the Hubbard model; there magnon = a hole + double occupation, while fat magnon = GG (indeed a hole) + GM (?); Two matrix (Z, W) quantum mechanics $\sim$ spinful fermion system like Hubbard?