Tachyons & solitons in the D1-D5 string

Jejjala, Madden, SFR & Titchener, hep-th/0504181

SFR, hep-th/0509066

Simon Ross

Centre for Particle Theory, U. of Durham

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1. Smooth geometries in the D1-D5 system
   - Review of supersymmetric solitons
   - New non-SUSY solitons
2. Winding tachyons in D1-D5 system
   - Winding tachyons in black strings
   - Tachyons with a twist
   - Solitons as endpoints of tachyon condensation
3. Discussion

Supersymmetric solitons

Balasubramanian, de Boer, Keski-Vakkuri & Ross; Maldacena & Maoz

Smooth soliton solutions

Compactification on $T^4 \times S^1_y$ to 5d AF solution

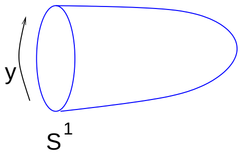
⋆ First example obtained as a special case of rotating black hole

$S^1 \to 0$ is $R\partial_y - \partial_\phi$.

Rotating BPS solution; Angular momentum fixed by charges

Topology

$T^4 \times \mathbb{R}^2_{r,y} \times \mathbb{R}_t \times S^3_{(\theta,\tilde{\phi},\psi)}$.



'Near-core' region *global* $\mathrm{AdS}_3 \times S^3 \times T^4$.

Dual CFT:

Global $\mathrm{AdS}_3 \times S^3 \leftrightarrow$ NS vacuum state

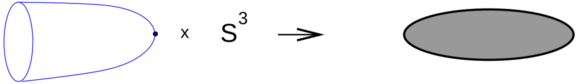
Twist $\tilde{\phi} = \phi + y/R$ corresponds to spectral flow.

Soliton identified with RR ground state of maximal R-charge.

Supertube picture

Mateos & Townsend; Lunin, Maldacena & Maoz

On Kaluza-Klein reduction, obtain an AF 5d solution singular on a disc. KK monopole at boundary of disc; no net monopole charge.



▷ Similar to supertube picture:

- In original example, F1-D0 bound states with angular momentum expand into a D2-brane tube.
- Here, D1-D5 bound states expand into a KK monopole tube.

⋆ Supertubes can have arbitrary profiles.

Generalizations

- More general solutions, determined by arbitrary profile $\vec{F}(y) \in \mathbb{R}^4$. Identify with general RR ground states.

Lunin & Mathur; Lunin, Maldacena & Maoz

- Relate to classification of SUSY solutions: general solutions have the form

$$ds^2 = H^{-1}(-(dt - A)^2 + (dy + B)^2) + Hd\vec{x}^2$$

Gutowski, Martelli & Reall

- 3-charge solutions: replace flat base by Gibbons-Hawking space. Fibration over $\mathbb{R}^3$: solutions labeled by locations of poles in $\mathbb{R}^3$.

Giusto, Mathur & Saxena; Bena & Warner; Berglund, Gimon & Levi

Philosophical interlude

Mathur, . . . (see e.g. hep-th/0502050)

Smooth geometries ↔ CFT microstates

Microscopic picture of black hole entropy arose from counting CFT microstates. Relation between these pictures?

Mathur *et al*: 'Fuzzball'

- "Horizon" arises by coarse graining
- Microstates described directly in geometric terms
- Evidence: probe scattering, counting states in supertube picture.
- Problems: finding enough solitons, large curvatures

No real black hole - no information loss problem

Philosophical interlude

Black hole effective description (Balasubramanian *et al*):

- Typical state well-approximated by a black hole for simple probes.
- Explicit analysis for SUSY states: superstar effective description
- Don't have enough control of generic non-SUSY states

⋆ Dynamical issues not yet addressed.
Forming a soliton in gravitational collapse difficult:
topology, horizon teleological.

Important to understand relation between geometry & CFT states in more detail, find more examples

Non-supersymmetric solitons

Look for soliton solutions in general "black string" metric.

- Metric involves two harmonic functions $H_{1,5}$, $g(r) = (r^2 + a_1^2)(r^2 + a_2^2) - Mr^2 = (r^2 - r_+^2)(r^2 - r_-^2)$. Coordinate singularities at $H_{1,5} = 0$, $r^2 = r_\pm^2$.
- If $r_+^2 > 0$, horizon; if $r_+^2 < 0$, conical singularity.
- Require $H_{1,5}(r_+) > 0$.
- Make $r^2 = r_+^2$ smooth origin: Need $||\xi||^2 = 0$ at $r^2 = r_+^2$ for some

$$\xi = R\partial_y + n\partial_\psi - m\partial_\phi,$$

$m, n \in \mathbb{Z}$.

Non-supersymmetric solitons

All parameters fixed in terms of Q_1, Q_5, R, m, n.

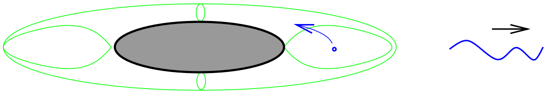
- Completely smooth non-BPS solutions.
- t is a global time function, so no CTCs.
- Can also consider $\mathbb{Z}_k$ orbifolds:
  Take $y \sim y + 2\pi Rk$ at fixed $\tilde{\phi}, \tilde{\psi}$ closed cycle.
- $m + n$ odd to have periodic fermions on asymptotic S^1_y.

CFT interpretation

Instability of the non-susy solitons

Cardoso, Dias, Hovdebo & Myers

Non-supersymmetric solitons have an ergoregion:



Ergoregion causes instability (Friedman):

- Modes localised in ergoregion have negative energy
- Time-dependent modes emit radiation to infinity; carries positive energy
- Localised mode will have increasingly negative energy: instability.

Non-supersymmetric solitons are classically unstable.

Tachyons in string theory

Many string backgrounds have tachyon excitations:

- Closed strings:
  - Bosonic string
  - Superstring compactified on a circle with supersymmetry-breaking boundary conditions, $R < \ell_s$.
- Open strings:
  - D-branes in bosonic string
  - $Dp - \bar{D}p$

Endpoint of decay?

⋆ Open strings: D-branes decay to flat space. Sen

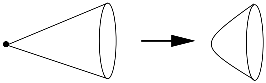
Can also get lower branes from non-trivial tachyon profiles.

Localized closed string tachyons

Adams, Polchinski & Silverstein, . . .

Progress on open strings motivated study of *localized* closed string tachyons:

- Orbifolds, NS5-branes, Twisted compactifications, . . .
- $\mathbb{C}/\mathbb{Z}_n$ orbifold has tachyons in twisted sectors
- Conjectured endpoint 'Decay to nothing': tachyon condensation removes the region of spacetime where the closed string was tachyonic.



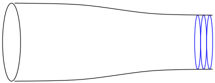
Studied by worldsheet RG flow, D-brane probes, grav solns.

Winding tachyons in black strings

Horowitz, hep-th/0506166

S^1 with antiperiodic fermions $\Rightarrow$ tachyon if $R < \ell_s$.

Localized tachyon: take Q_1 F1-strings wrapped on an S^1. Back-reaction shrinks S^1.



6d black string solution:

$$ds^2 = H_1^{-1}(r)[-f(r)dt^2 + dy^2] + f^{-1}(r)dr^2 + r^2 d\Omega_3^2.$$

Horizon at $r = r_0$, $f(r_0) = 0$. $Q_1 = r_0^2 \sinh 2\alpha$. $y \sim y + 2\pi R$.

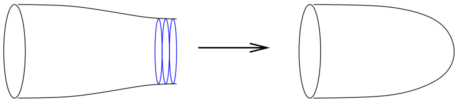
$\star$ Expect tachyon outside horizon if $\ell_s \cosh \alpha > R$.

$\triangleright$ Endpoint?

Winding tachyons in black strings

Horowitz, hep-th/0506166

Decay to nothing:



There's a static bubble, obtained by double analytic continuation of black string:

$$ds^2 = H_1^{-1}(r)[f(r)dy^2 - dt^2] + f^{-1}(r)dr^2 + r^2 d\Omega_3^2.$$

Larger bubbles expand, smaller bubbles contract.
Static bubbles always have $Q/R^2 < 1$.
Horowitz showed strings with $Q/R^2 < 1$ decay to static bubbles; strings with $Q/R^2 > 1$ decay to expanding bubbles.

New endpoint for Hawking radiation.

▷ Antiperiodic bc problematic: Casimir energy, bubble decay.

Winding tachyons in D1-D5 black strings

How can we have tachyons? Periodic bc on S^1.

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Consider a twisted circle, orbits of $\xi = R\partial_y - m\partial_\phi$,
i.e., S^1 parametrized by y at fixed $\tilde{\phi} = \phi + ym/R$. Note $m \in \mathbb{Z}$.

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2,$$

so fermions antiperiodic around ϕ.

$\Rightarrow$ For m odd, fermions antiperiodic around S^1_ξ.

$\star$ Can choose parameters so $||\xi||^2 < \ell_s^2$ outside hor, curv small:

- Fixes m in terms of charges,
- $M - a_1^2$ small.
- Presence of $dyd\phi$ cross terms in metric crucial.
- Can also make circle small over a large radial proper distance.

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Near-horizon limit

work in progress: Goodsell, Rangamani, SFR

Near-horizon limit is a BTZ black hole, $\mathrm{AdS}_3/\mathbb{Z} \times S^3_{\theta,\tilde{\phi},\psi}$.

▷ Know something about strings on AdS_3 with NS fields:

Maldacena & Ooguri, Natsuume & Satoh, Hemming & Keski-Vakkuri, ...

⋆ Partition function for the bosonic string on Euclidean BTZ studied by Maldacena, Ooguri & Son, hep-th/0005183.

- Determined spectrum of strings on AdS
- Long strings winding S^1 at infinity
- We want to see spectrum on BTZ; re-interpret their partition function, extend to superstring
- Expect to see tachyon in winding/long string sectors

⇒ Read off spectrum of strings; quantify localization of tachyon near horizon.

Solitons as endpoints of tachyon condensation

Conjecture soliton is always the endpoint:

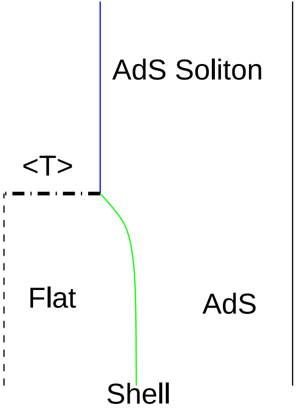
- $\exists$ soliton for any Q_1, Q_5, R, m (unlike earlier black string)
- ADM mass of soliton $<$ ADM mass of tachyonic black string
- S^3 at $r = r_+ \sim S^3$ at horizon.

$\star$ Good that we don't see expanding bubble: spacetime is asymptotically SUSY.

Localized tachyon in a collapsing shell

Horowitz & Silverstein

Interesting new example: Consider D3-branes distributed on an S^5. Identify a direction along D3-branes with antiperiodic boundary conditions for fermions.



- Tachyon if $L < \ell_s$ at shell
- Everywhere weak curvature
- Give shell small velocity, tachyon appears at some time
- Natural endpoint AdS soliton: requires non-trivial relaxation.
- Model for tachyon inside black hole horizon

Discussion

- Explicit map between CFT states & soliton geometries
  - Includes *non-SUSY* solitons
  - Understand emergence of spacetime?
  - Coarse-graining & black hole solutions
- Black strings nice egs of localized closed string tachyons:
  - Asymptotically susy, localized decay
  - Curved geometry, need to fine tune parameters
- Decay to solitons: new connection between black strings and solitons

Discussion

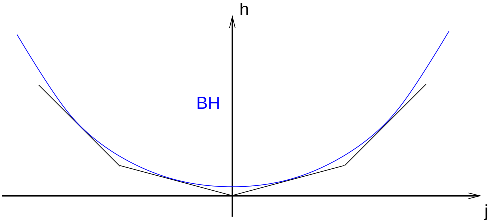
Future issues:

- Look for more general solitons: non-SUSY
- Understand instability of soliton solutions in brane terms
- Quantitative study of tachyon in near-horizon region: BTZ black hole
- Tachyon condensation: explore worldsheet RG, D-brane probes
- Study tachyon in T-dual theory

D1-D5 CFT

Dual description in terms of $1+1$ CFT on $\mathbb{R} \times S^1$:

- CFT deformation of σ-model on $(T^4)^{Q_1 Q_5}/S_{Q_1 Q_5}$, $c = 6 Q_1 Q_5$.
- $SO(2,2) \times SO(4)_R = SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SU(2) \times SU(2)$ symmetry, charges (h, j), $(\bar{h}, \bar{j})$
- NS, R sectors; related by spectral flow.



Focus on Ramond sector: periodic fermions on S^1.

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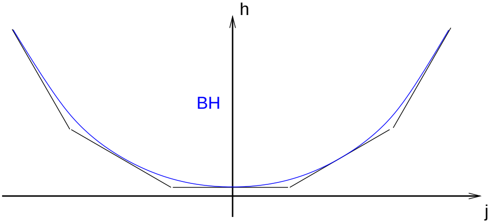

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CFT intepretation of solitons

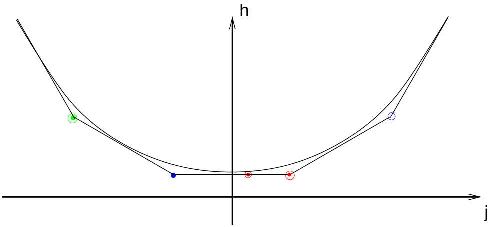
Near-core AdS region: $\mathrm{AdS}_{3(t,y,r)} \times S^3_{(\theta,\tilde{\phi},\tilde{\psi})} \times T^4$

$\tilde{\phi} = \phi + ym/R$, $\tilde{\psi} = \psi - yn/R$.

Read off CFT charges from asymptotics:

$$h = \frac{c}{24}(m+n)^2, \quad j = \frac{c}{12}(m+n),$$

$$\bar{h} = \frac{c}{24}(m-n)^2, \quad \bar{j} = \frac{c}{12}(m-n),$$



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Read off CFT charges from asymptotics:

$$h = \left(1 + \frac{(m+n)^2 - 1}{k^2}\right), \quad j = \frac{c}{12}\frac{(m+n)}{k},$$

$$\bar{h} = \left(1 + \frac{(m-n)^2 - 1}{k^2}\right), \quad \bar{j} = \frac{c}{12}\frac{(m-n)}{k},$$