Can Time End?

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Goal of this Talk

- To describe the physics of cosmological singularities
- To come closer to an understanding of the origin and fate of space-time

Based on the following papers:

hep-th/0603104, Daniel Robbins, Emil Martinec + S. S.

hep-th/0601062, Ben Craps, Arvind Rajaraman + S. S.

which further develop the models appearing in:

hep-th/0509204, Daniel Robbins & S. S.

hep-th/0506180, Ben Craps, Erik Verlinde & S. S.

There are two models that we will consider today. Both are generalizations of the original BFSS Matrix model. Both are null cosmologies.

- Matrix Big Bang – description of a light-like linear dilaton in type IIA string theory

I want to describe the leading quantum mechanical effects in both models.

- Null-Brane Matrix model – description of the null-brane “funnel” space-time in M-theory

I Matrix Big Bang

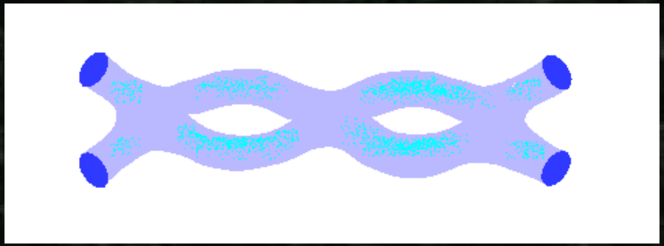
The space-time theory is the light-like linear dilaton.

$$g_s = e^{\phi}, \qquad \phi = -QX^+$$

The string metric is flat while the Einstein metric sees a “crunch”

$$ds_E^2 = e^{QX^+/2}\, ds_{10}^2.$$

Big Bang



Strong string coupling

Weak coupling

$X^+ = -\infty$

$X^+ \to \infty$

No a priori definition of string theory on this background.

$$\delta\lambda \sim \Gamma^{+}\partial_{+}\phi\epsilon \quad \Rightarrow \quad \Gamma^{+}\epsilon = 0.$$

Background preserves 16 supersymmetries.

At strong coupling, lift to M-theory with a metric and curvatures:

$$R_{+11+11} \sim e^{-4QX^{+}/3}, \qquad R_{+i+i} \sim e^{2QX^{+}/3}.$$

Divergent tidal forces.

We will use Matrix theory to provide a definition.

On decoupling, we find a non-perturbative definition of this background in terms of Matrix strings on the Milne orbifold,

$$ds^2 = e^{2Q\tau}(-d\tau^2 + d\sigma^2), \qquad \sigma \sim \sigma + 2\pi\ell_s.$$

When is decoupling valid?

This is a theory of D-strings in a specific IIB background.

$$\ell_s^{eff} \sim \frac{1}{\sqrt{\epsilon}} e^{-\epsilon Q X^+/2}$$

We care about energies $E_{typ} \sim \epsilon E^-$ so string oscillators decouple if

$$\epsilon E^- \ell_s^{eff} \sim \sqrt{\epsilon} E^- e^{-\epsilon Q X^+/2} \to 0.$$

Similarly, gravity decouples if

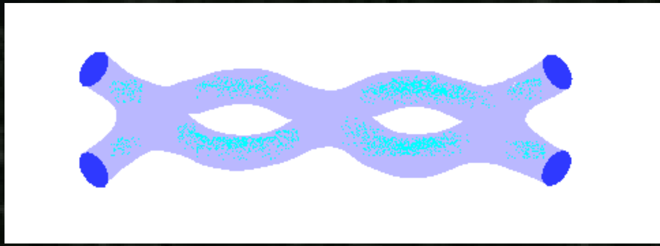
$$G_N^{eff} (\epsilon E^-)^8 \sim \epsilon^6 (E^-)^8 e^{-2\epsilon Q X^+} \to 0 \qquad \text{as } \epsilon \to 0.$$

$$S = \frac{\ell_s^2}{2\pi} \int \left(\frac{1}{2}(D_\mu X^i)^2 + \bar{\psi} \not{D} \psi + e^{-2Q\tau} \pi^2 F_{\mu\nu}^2 - \frac{1}{4\pi^2} e^{2Q\tau} [X^i, X^j]^2 \right.$$
$$\left. + \frac{1}{2\pi} e^{Q\tau} \bar{\psi} \gamma_i [X^i, \psi] \right).$$

We identify $\hbar$ with $\frac{1}{\ell_s^2}$.

The value of Q has no invariant physical meaning. It is either zero or non-zero.

Matrix Big Bang



Weakly coupled Yang-Mills

Strongly coupled
Matrix strings

$X^+ = -\infty$

$X^+ \to \infty$

RG flow corresponds to time evolution.

The singularity is replaced by a non-abelian gluon phase.

$$g_{YM} = \frac{1}{g_s \ell_s}$$

Let us examine the leading quantum mechanical effects.

Remarkably little is known about interacting field theories on time-dependent spaces.

We would like to define “effective physics” at low-energies. However, the usual Wilsonian procedure always breaks down at sufficiently early times.

Expand around a vacuum: $\mathrm{Tr}(X)^2 \sim b^2.$

$M_W^2 \sim e^{2Q\tau} b^2$

Always below the cut-off for sufficiently early times.

Characteristic break down time: $\tau_{\text{non-abelian}} \sim \frac{1}{Q} \ln(\Lambda/b)$

At this time, the theory is non-abelian. We will see that the 1PI effective action does appear to make sense.

On the other hand, perturbation theory breaks down at late times,

$$g_{YM}/b \sim 1, \qquad \tau_{string} \sim \frac{1}{Q} \ln(\ell_s b)$$

At this point, perturbative string theory should take over.

Supersymmetry is broken: does Matrix theory make sense?
Do we have flat directions?

The Milne orbifold is non-Hausdorff. Does this lead to problems in defining field theory? etc.

We will see that there appear to be no problems in defining and studying the quantum mechanics of this theory.

Further, the theory appears to be more effective computationally than we perhaps had a right to expect.

First one can check that integrating out particles with a time-varying mass does not introduce non-localities in the 1PI effective action.

Let’s take that as given.

How do we compute quantum effects?

We will use the description of the Milne space as an orbifold of flat space.

$$\xi^{\pm} = \frac{1}{\sqrt{2}Q} e^{Q(\tau \pm \sigma)} \longrightarrow \mathrm{ds}^2 = -2 d\xi^+ d\xi^-$$

Boost identification: $\xi^{\pm} \sim \xi^{\pm} e^{\pm 2\pi Q \ell_s}$.

In this frame, particles of non-zero spin are not periodic under the identification.

A boost invariant wavefunction satisfies:

$$\phi_s(e^{2\pi Q \ell_s} \xi^+, e^{-2\pi Q \ell_s} \xi^-) = e^{2\pi Q \ell_s s} \phi_s(\xi^+, \xi^-).$$

The leading quantum mechanical effect should be a potential for the impact parameter, b.

So we want to compute a time-dependent version of the Coleman-Weinberg potential.

Integrate out quadratic fluctuations. This generates a determinant. For example, for a massive boson we want to evaluate:

$$\det{}^{(-1/2)}(H) = \det{}^{(-1/2)}\left(2\frac{\partial^2}{\partial\xi^+\partial\xi^-} + b^2\right)$$

$$\det^{(-1/2)}(H) = \exp\left(\frac{1}{2}\int d^2\xi \int dt\, \frac{e^{-it(H-i\epsilon)}}{t}(\xi,\xi)\right)$$

Expressed in terms of the heat kernel.

Usually SUSY guarantees that all contributions to the potential vanish but not here!

$$\mathrm{e}^{-itH_s}(\xi,\xi) =$$

$$-\sum_n \frac{1}{(2\pi)2t} \exp\left(-itb^2 - i\frac{\xi^-\xi^+}{2t}(1-e^{2\pi Q\ell_s n})(1-e^{-2\pi Q\ell_s n}) + 2\pi Q\ell_s ns\right)$$

The UV divergences from small t are regularized.

Summing over the different helicities (ghosts, fermions, gauge-fields and scalars).

$$\int V_{eff}(b,\xi^+,\xi^-) \sim$$

$$-\int d^2\xi \, \sum_{n=-\infty}^{\infty} \sqrt{\frac{b\sinh^4(\pi Q\ell_s n/2)}{2\sinh^2(\pi Q\ell_s n)\xi^+\xi^-}} \times K_1(\sqrt{8b^2\sinh^2(\pi Q\ell_s n)\xi^+\xi^-}).$$

$$K_1(z) \sim \frac{1}{\sqrt{z}}e^{-z}, \quad z >> 1$$

$$K_1(z) \sim \frac{1}{z}, \quad z << 1$$

The potential lifts the flat directions.

Late times:

$$\int \sqrt{g} V_{eff}(b) \sim \int d\sigma d\tau \sqrt{\frac{b}{g_s}} e^{-\frac{Cb}{g_s}}.$$

D-brane induced potential that decays super-fast!

Early times:

$$\int \sqrt{g} V_{eff} \sim \int d^2\xi \, \frac{1}{\xi^+\xi^-} \log(2b^2\xi^+\xi^-).$$

Attractive potential between eigenvalues. So supersymmetry and the flat directions are restored at late times.

A final comment about general covariance.

We should be able to couple Matrix strings to gravity without any local anomalies. So the breaking of Poincare invariance is spontaneous.

$$S_{eff} = \int \sqrt{\tilde{g}} \left\{ \partial_\mu \Phi \partial^\mu \Phi + V^{(0)}(\Phi) + R(\tilde{g}) V^{(1)}(\Phi) + \ldots \right\},$$

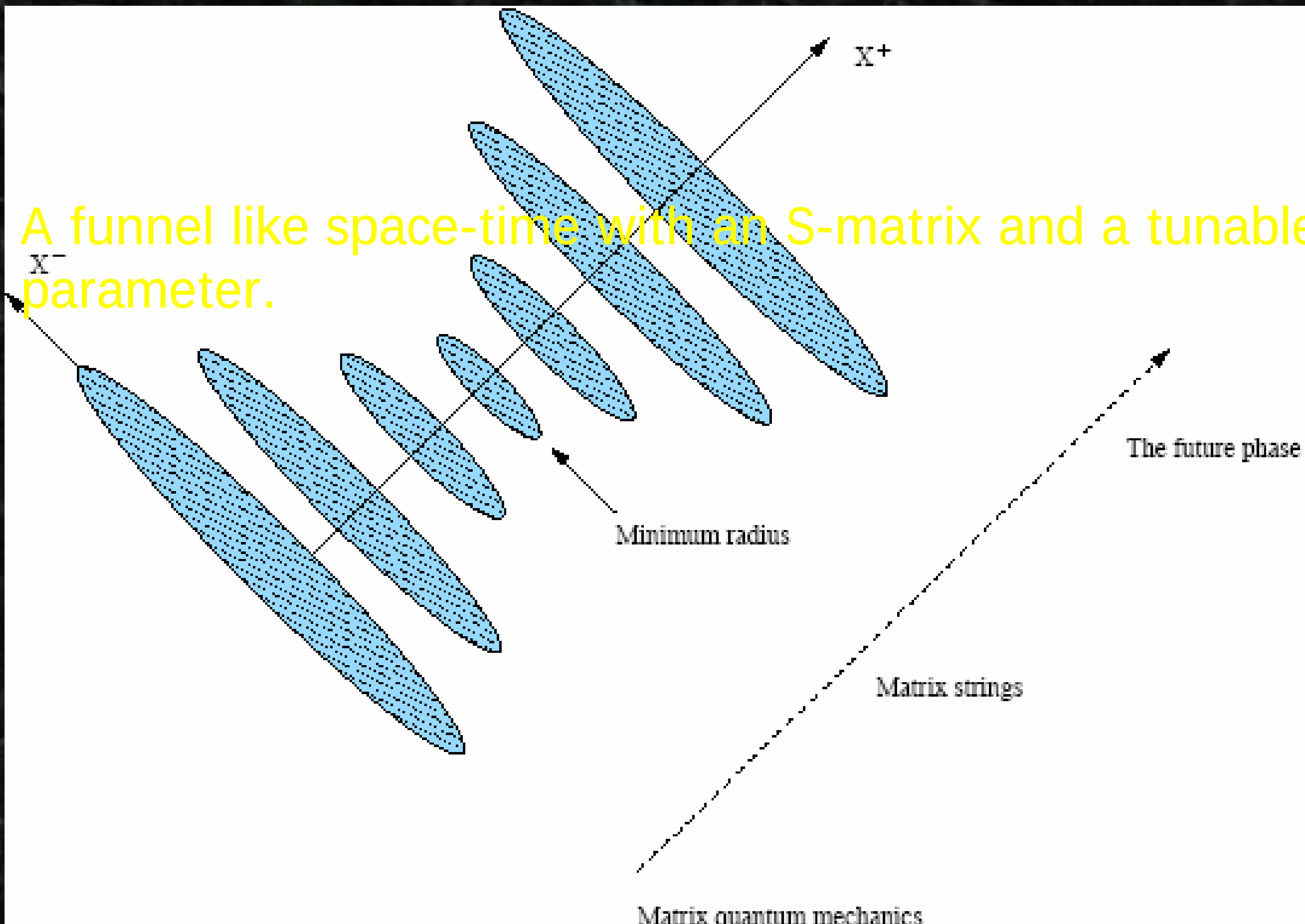
This is a violation of standard lore from Wilsonian RG.

This is not the form of the potential we found!

This suggests a mixing of high and low scales.

II Null-brane Matrix Model

A funnel like space-time with an S-matrix and a tunable parameter.



Orbifold description:

$$\mathrm{X} = \begin{pmatrix} x^+ \\ x^- \\ x \\ z \end{pmatrix} \rightarrow \quad g \cdot X = \begin{pmatrix} x^+ \\ x^- + 2\pi\lambda x + 2\pi^2\lambda^2 x^+ \\ x + 2\pi\lambda x^+ \\ z + 2\pi L \end{pmatrix}.$$

Depends on (λ, L).

Can again compactify $X^- \sim X^- + 2\pi R,$

and consider the Matrix description.

This is the theory of an array of boosted D0-branes.

$$S \sim \int d\tau d\sigma \, \mathrm{Tr} \Big((D_0 x^i)^2 + (D_0 x)^2 - L^2 (D_1 x^i)^2 - L^2 (D_1 x)^2$$

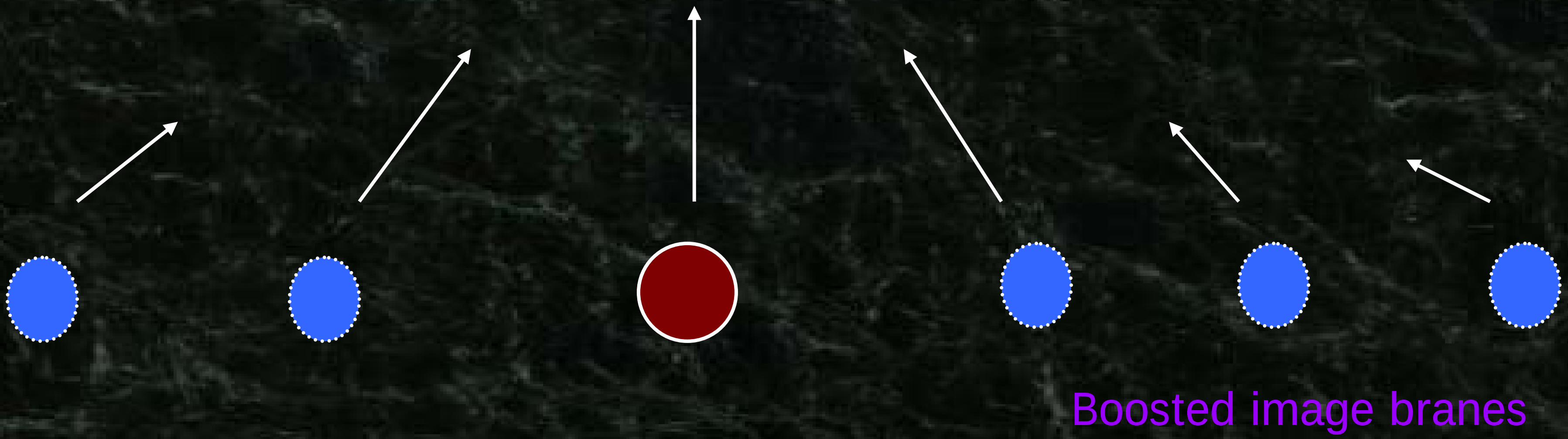
$$- \sqrt{2} \lambda (x - \tau D_0 x) F + (L^2 + \tfrac{1}{2} \lambda^2 \tau^2) F^2 + \tfrac{1}{2} \left[x^i, x^j \right]^2$$

$$+ \left(\left[x, x^i \right] + \tfrac{i \lambda \tau}{\sqrt{2}} D_1 x^i \right)^2 \Big)$$

$\tau = L = 0$ is the crunch.

A new flat direction opens up where sigma excitations become light.

$$(\mathrm{L}^2 + \tfrac{1}{2} \tau^2)(D_1 x^i)^2 \;\rightarrow\; 0.$$



D-branes will interact via their velocity-dependent interaction which will generate a static potential.

Classically, the brane and its images will collapse into a black hole whose size grows polynomially with 1/L.

(Horowitz & Polchinski)

A comment on the large N limit:

Relative velocity of D0-branes related by g^n is $2\pi n\lambda$.

View lambda as a velocity. The kinetic energy of N D0-branes should scale as 1/N^2.

$$\text{K.E.} \sim v^2/R \sim 1/N^2 \quad \Rightarrow \quad v \sim 1/N \text{ and } \lambda \sim 1/N.$$

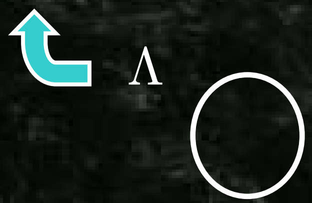
We can only well approximate a black hole with S ~ N. This determines a lower bound on N:

$$\mathrm{N}_{min} \sim \left(\frac{\ell_p^3 p^+}{L^3}\right)^{\frac{d-1}{d-4}}$$

d non-compact dimensions.

The potential is computed in much the same way as before except that the operators are nicer.

$$\exp\left(\int d\tau d\sigma\, V_{\text{eff}}\right) = \det{}^{-1/2}(H) = \det{}^{-1/2}\left(-\partial_\tau^2 - n^2(L^2 + \tau^2/2)\right).$$



$$\det{}^{-1/2}(H) = \exp\left(\tfrac{1}{2}\int d\tau d\sigma \int \tfrac{dt}{t} e^{itH}(\tau,\tau)\right)$$

The heat kernel is the SHO kernel given by Mehler's formula:

$$\mathrm{e}^{itH}(\tau,\tau) = e^{in^2L^2t}\left(\frac{in}{2\pi\sqrt{2}\sinh(\sqrt{2}nt)}\right)^{1/2} \times$$

$$\exp\left[\frac{in\lambda}{\sqrt{2}\sinh(\sqrt{2}nt)}\left(\tau^2(\cosh(\sqrt{2}nt) - 1)\right)\right]$$

Finally we rotate the result to Euclidean space (simplified):

$$\mathrm{V}_{\mathrm{eff}} \sim \int \frac{dt}{t} \sum_{n=0}^{\infty} \sinh^4\left(\frac{n\lambda t}{2\sqrt{2}}\right) K(t, \lambda\tau, n)$$

where

$$\mathrm{K}(\mathrm{t},\tau, n) = e^{-(n^2L^2+b^2)t}\left(\frac{n\lambda}{2\pi\sqrt{2}\sinh(\sqrt{2}n\lambda t)}\right)^{1/2} \exp\left[-\frac{n\lambda\tau^2}{\sqrt{2}}\tanh\left(\frac{n\lambda t}{\sqrt{2}}\right)\right].$$

There is an infra-red divergence when

$$\mathrm{n}^2L^2 + b^2 \lesssim n\lambda/\sqrt{2}.$$

This signals a breakdown in the one-loop approximation.

The UV contribution to the potential comes from small proper times,

$$2\pi V_{\rm eff} \sim \int \frac{ds}{s}\, s^4 \sum_{n=1}^{\infty} e^{-n\Lambda s/\lambda} \left(\frac{n\lambda}{\pi s}\right)^{1/2}$$

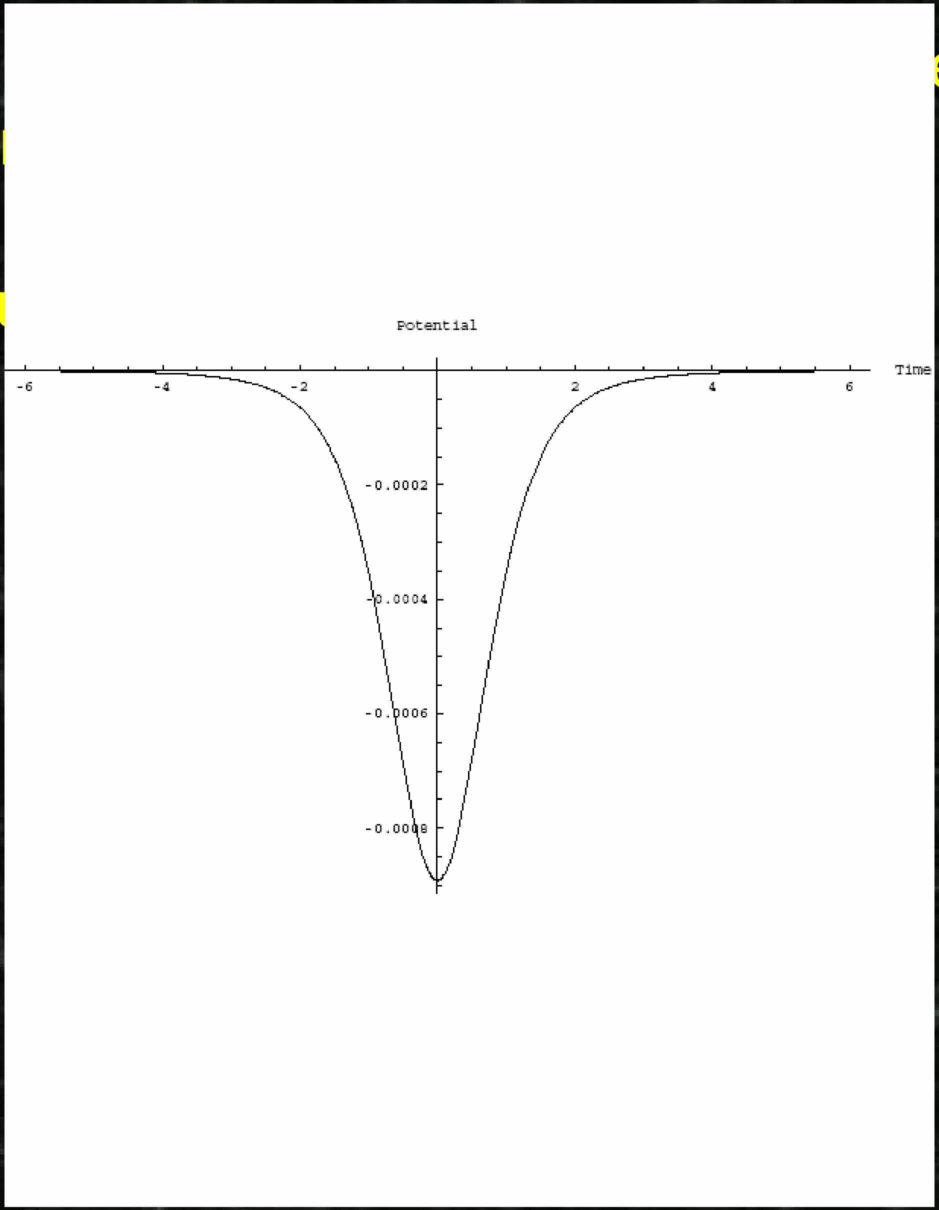
$$\sim \frac{1}{\sqrt{\pi}}\zeta(3)\Gamma(7/2)\frac{\lambda^4}{\Lambda^{7/2}}$$

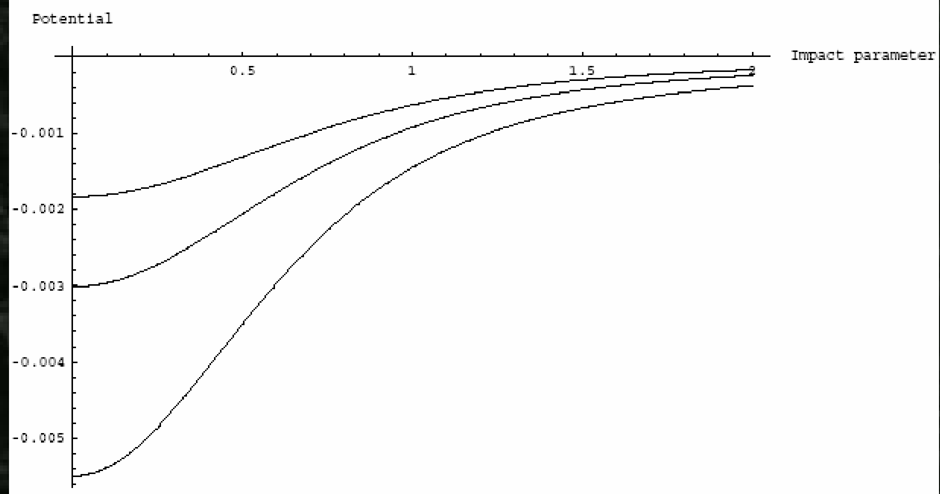
← v^4/r^7 DKPS

$s = nt \qquad \Lambda = L^2 + \frac{1}{2}\tau^2.$

The potential e leading velocity-deper

Plotted as a fu




Potential
Impact parameter
0.5
1
1.5
2
-0.001
-0.002
-0.003
-0.004
-0.005

The well becomes infinitely deep as L approaches the tachyon instability.

It tentatively appears we never escape from the non-abelian gluon phase and so conventional space and time end.

I haven’t yet discussed particle production but it is encoded in the imaginary part of the phase shift.

Holography ⟷ **Time-dependent Field theory**

Steps toward an understanding of:

- The origin and fate of space and time
- The definition of string theory in cosmological backgrounds
- A theory of quantum cosmology and initial conditions

A brief look at particle production. The linearized dynamics of the bosonic fields is solved by parabolic cylinder functions,

$$[-\partial_\tau^2 - \omega^2\tau^2]\psi_\nu(\tau) = \nu\psi_\nu(\tau)$$

$$\psi_\nu = \alpha\,\frac{e^{-\mu\pi/4}}{(2\omega)^{1/4}}\;D_{-i\mu-\frac{1}{2}}\left(e^{\frac{i\pi}{4}}\sqrt{2\omega}\,\tau\right)\;+\;\beta\,\frac{e^{-\mu\pi/4}}{(2\omega)^{1/4}}\;D^*_{-i\mu-\frac{1}{2}}\left(e^{\frac{i\pi}{4}}\sqrt{2\omega}\,\tau\right)$$

$$|\alpha|^2 - |\beta|^2 = 1 \qquad\qquad \mu = \frac{\nu}{2\omega} = \frac{nL^2}{\sqrt{2}}$$

Probability amplitude for particle production:

$$|\beta| = e^{-\pi\mu} = \exp\left[-\frac{\pi nL^2}{\sqrt{2}\lambda}\right].$$

Mode production becomes of order 1 when the effective coupling is of order one. Again a signal of the breakdown of perturbation theory.

$L \to 0$ with fixed λ yields a non-uniform large N limit.

Scaling lambda ~ 1/N suppresses particle production.

What about the production of multi-particle excitations corresponding to strings that thread of order N D0-branes?