

# Field/String Duality

Charles Thorn

*Department of Physics  
University of Florida*

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## 2 Introduction

Duality probably a misnomer: quantum field theory is well-formulated whereas string theory is not. For now the QFT side is the fundamental one.

- All we know for sure about string theory is its perturbation expansion in terms of worldsheet path integrals.
- The underlying theory for string is debatable: is it M theory? If so, what is M-theory?
- No such controversy for QFT: there is a Lagrangian which produces perturbation theory and lattice gauge theory gives it nonperturbative meaning.
- One can quite generally interpret the sum of planar diagrams for QFT as a worldsheet dynamical system.

For String Theory: sheds light on its foundation.

- IIB superstring on  $\text{AdS}_5 \times \text{S}^5$  is identical to  $\mathcal{N} = 4$  SUSY Yang-Mills
- More general AdS/CFT connections formulate string on other backgrounds as other Poincare invariant QFT's
- String on some backgrounds might be based on Poincare non invariant QFT's (e.g. Galilei invariant string bit models)

For Quantum Gravity: suggests a path for inducing gravity

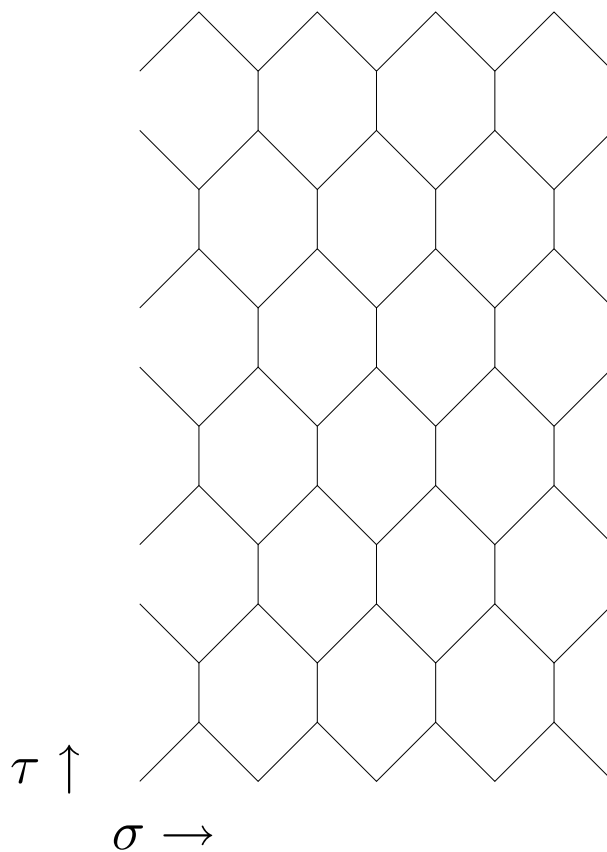
- An attractive notion (Sakharov, Adler, Zee, ...) for dealing with obstacles to quantizing gravity has been to imagine the graviton as a massless spin two bound state in QFT.
- Seems squelched by no-go theorem proved by Weinberg and Witten: Poincare invariant QFT can't produce massless spin 2 bound states.
- Field/String duality is typically holographic: string moves in higher dimensional space than that of QFT. The associated quantum gravity induced by the field theory should also be holographic, evading the no-go theorem.
- Quantum gravity in 4 space-time dimensions might be a QFT in 3 or less space-time dimensions. ('t Hooft's hologram)

For Quantum Field Theory: Field/String duality suggests that free string is a solution to certain field theories in the  $N_c \rightarrow \infty$  limit.

- Quark confinement, though well-supported numerically, needs a fundamental understanding.
- Open string can model a gluonic flux tube
- Formulating  $N_c = \infty$  QCD as a free string theory could be a step toward this goal.
- N. B. A string dual *does not by itself* establish confinement. (cf. AdS/CFT).

### 3 History

In 1970 Nielsen and Olesen and, independently, Sakita and Virasoro proposed that string-string scattering amplitudes (a.k.a. dual resonance models) were approximations to sums of large planar Feynman diagrams (“fishnets”):



Raises some questions:

- Need to postulate gaussian propagators to actually derive the amplitudes from this idea.
- How control planar approximation?
- How justify dominance of large fishnet diagrams?

In 1974 't Hooft observed that  $N_c \rightarrow \infty$ ,  $g_s^2 N_c$  fixed singled out the planar diagrams of an  $SU(N_c)$  gauge theory. This limit dramatically suppresses physics complications if confinement holds:

- Hadrons are stable free particles.
- Leading order scattering amplitudes show only pole singularities in the invariants  $p_i \cdot p_j$ .
- Localized quark-antiquark sources at separation  $L \gg \Lambda_{QCD}^{-1}$  form unbreakable gluonic flux tube so  $U(L) \sim T_0 L$ .

But to get fishnets also need  $g_s^2 N_c \gg 1$  (CBT, 1978).

For QCD  $g_s$  is momentum dependent and not a tunable parameter. There may be no scale where  $g_s^2 N_c$  is very large, which suggests string theory could have only a very rough qualitative relation to QCD.

Or: If  $N_c \rightarrow \infty$  *defines* QCD string, then it is very different from the string of string theory.

In 1997 Maldacena proposed [2]:

$N_c = \infty$ ,  $\mathcal{N} = 4$  super  $SU(N_c)$  gauge theory  
equivalent to  
*free* IIB superstring theory on  $AdS_5 \times S_5$

First example of what is now known as the AdS/CFT  
correspondence.

The CFT (Conformal Field Theory) here is a scale  
invariant (finite) quantum field theory.

Field/String Duality is more general.

The essential point seems to be  $N_c \rightarrow \infty$ :

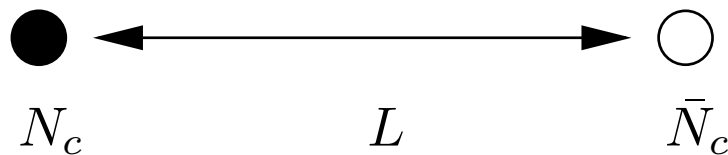
- At  $N_c = \infty$  perturbation theory is sum of only planar diagrams, suggesting the worldsheet of a string description for *any*  $N_c \times N_c$  matrix field theory.
- The AdS/CFT correspondence is most sharply formulated in the  $N_c \rightarrow \infty$  limit.
- In the regime  $g_s^2 N_c \gg 1$  the AdS string is accurately described semi-classically. We probably won't have the luxury of such an approximation for QCD: its dual string must be inherently quantum.

## 4 Real QCD v.s. $\mathcal{N} = 4$ Super Yang-Mills

- QCD has a mass gap and confinement.
- QCD has scale dependent coupling: therefore no free parameters with massless quarks.
- $\mathcal{N} = 4$  super Yang-Mills is conformally invariant: no mass gap, no confinement, but a fixed coupling.
- For  $\mathcal{N} = 4$ , weak and strong coupling have meaning for all processes.
- For QCD, weak coupling only at high momentum and stronger coupling at low momentum arbitrarily strong coupling may never be reached.

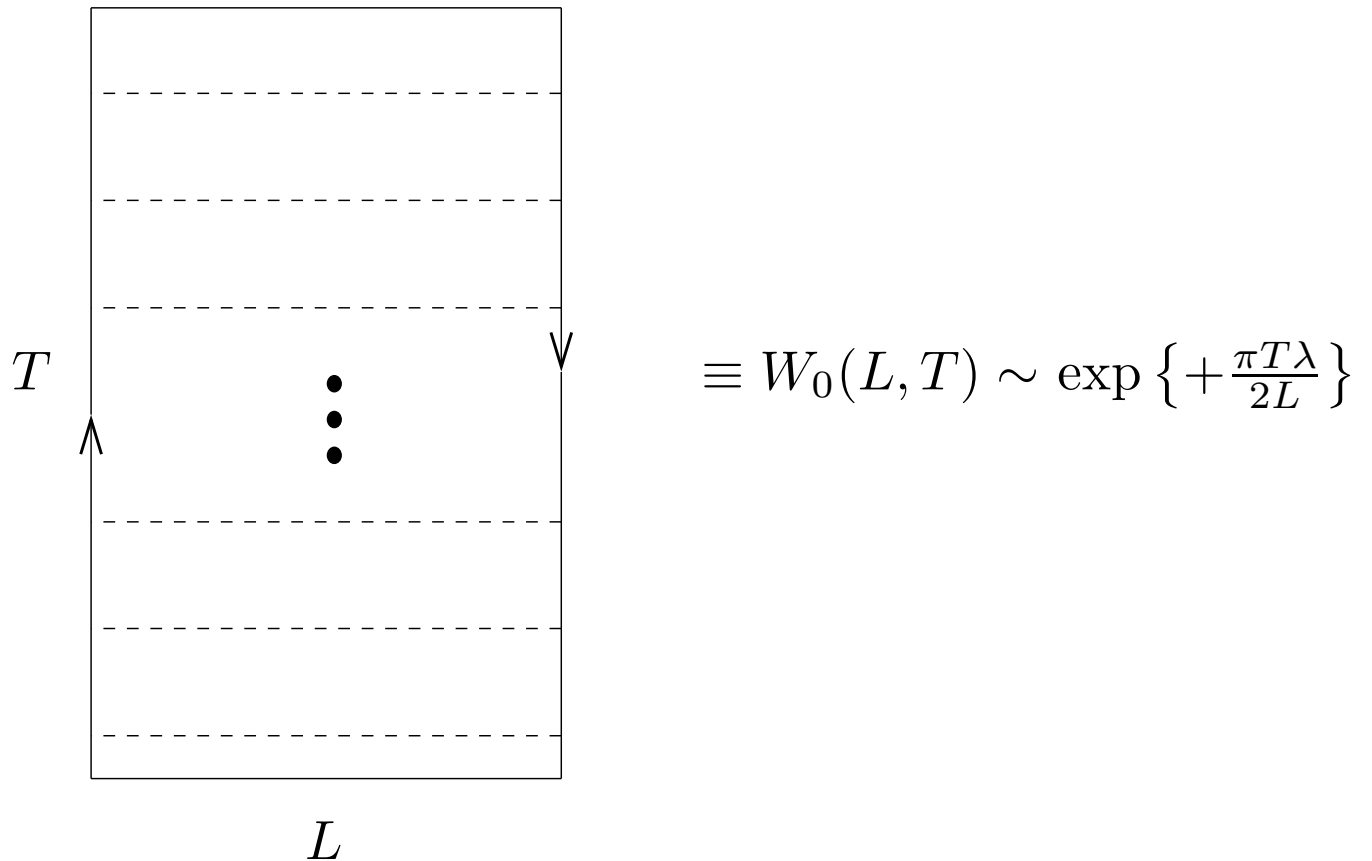
# 5 Separated $Q\bar{Q}$

A good test of confinement in gauge theory is its response to separated static color sources: a time-like rectangular Wilson Loop.



$$\begin{aligned}
 \langle W(L, T) \rangle &\underset{T \rightarrow \infty}{\sim} e^{-E(L)T} \\
 &\underset{L, T \rightarrow \infty}{\sim} e^{-T_0 L T} && \text{for confinement} \\
 &\underset{L, T \rightarrow \infty}{\sim} e^{-cT/L} && \text{conformal invariance}
 \end{aligned}$$

## 6 $Q\bar{Q}$ at Weak Coupling

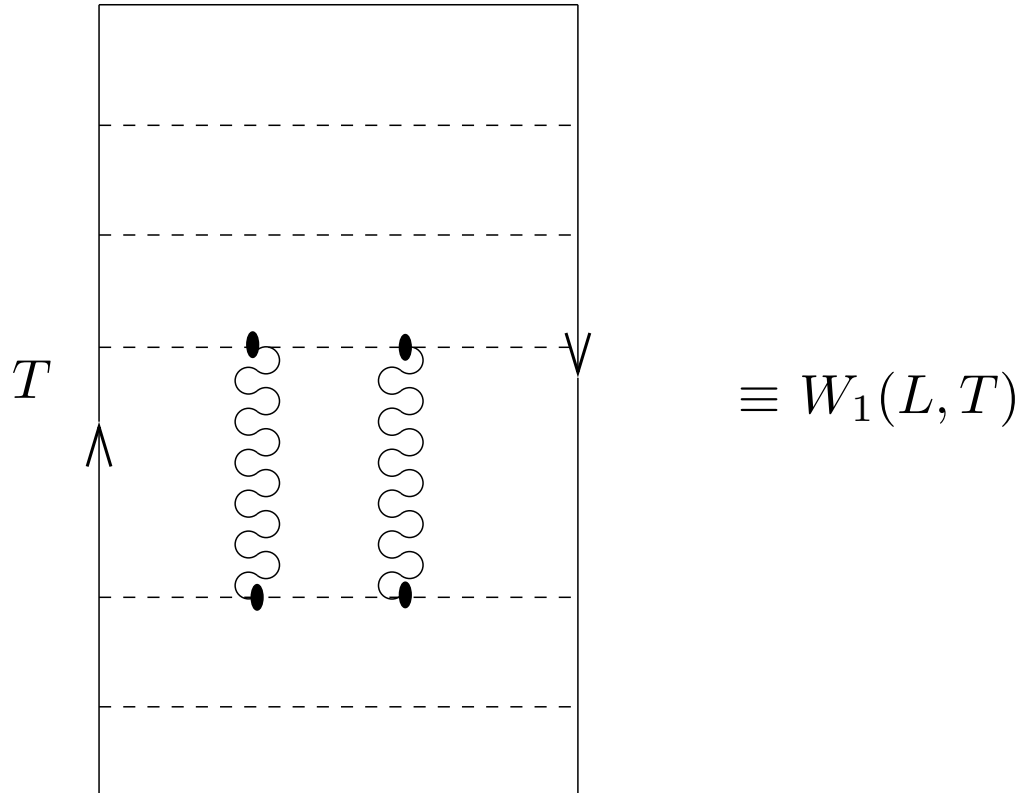


$$\lambda = \frac{N\alpha_s}{\pi}$$

- The Wilson loop ensures  $Q\bar{Q}$  are in a singlet.
- The above behavior shows  $E_{singlet} = -\pi\lambda/2L$ .
- This approximation shows only one eigenstate.

$$R_0(E, L) \equiv \int_0^\infty dT e^{ET} W_0 \sim \frac{\rho_0}{E + \pi\lambda/2L}$$

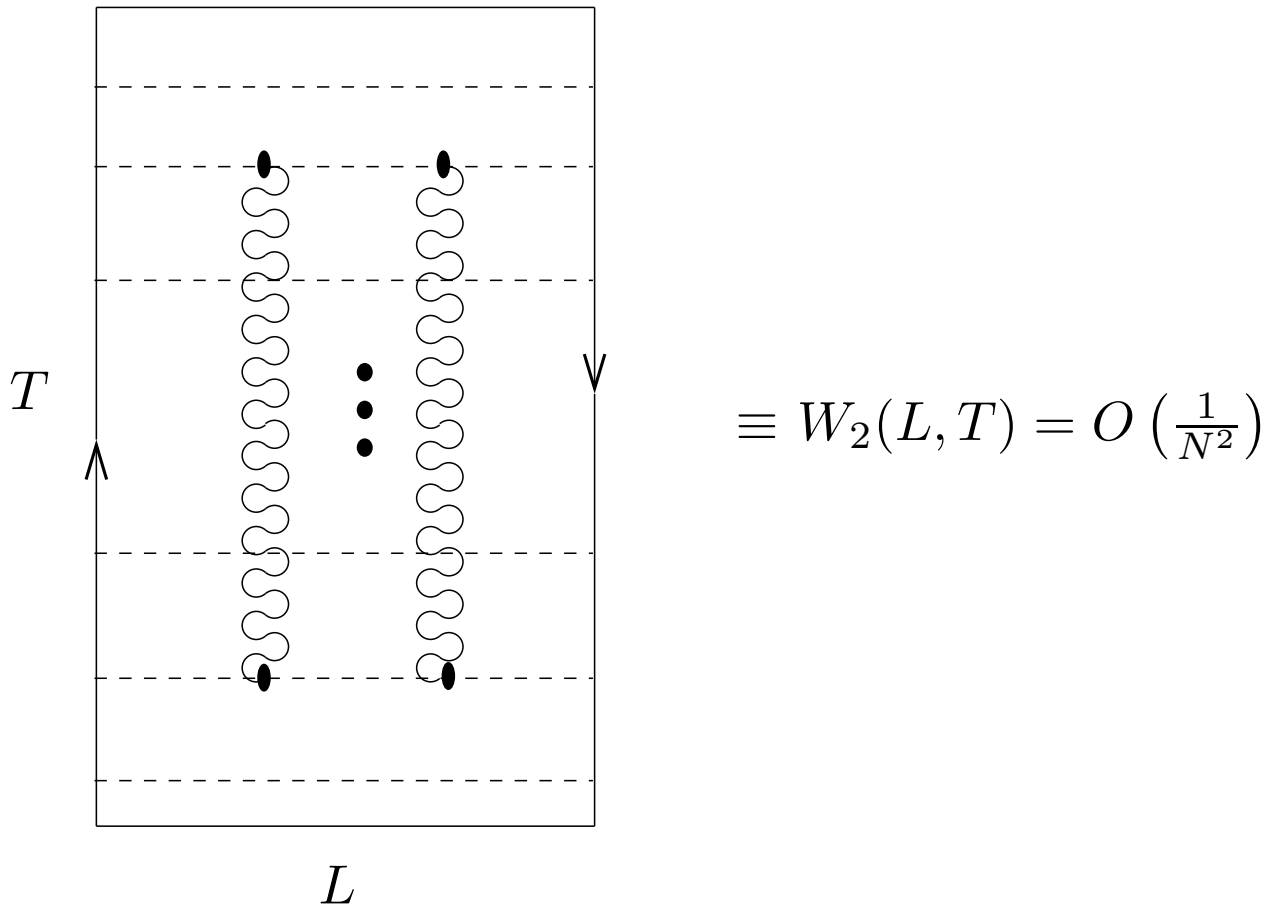
## A Planar Radiative Correction



$$R_1(E, L) \sim \frac{\rho'_0}{E + \pi\lambda/2L} + \int_0^\infty dE' \frac{\rho_1(E')}{E' - E}$$

- Only the continuum with  $E > 0$  couples to planar Wilson loop.

# A Nonplanar Radiative Correction



$$R_2(E, L) \sim \frac{1}{N^2} \int_{-\pi\lambda/2L}^{\infty} dE' \frac{\rho_2(E')}{E' - E}$$

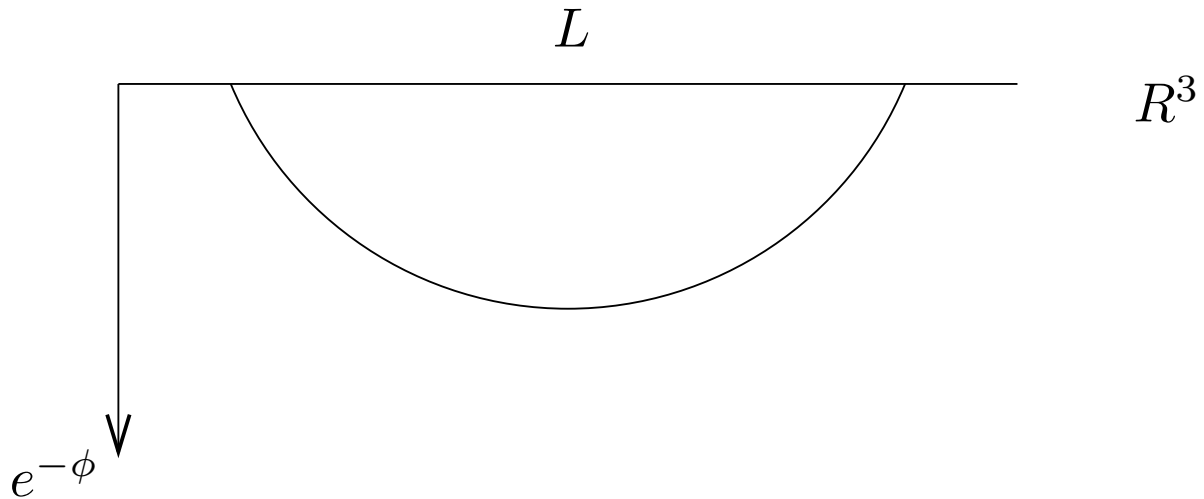
At  $N = \infty$  there is a gap in the  $Q\bar{Q}$  system.  
 (Klebanov, Maldacena, Thorn [6])

## 7 $\mathcal{N} = 4$ at Strong Coupling

For  $\mathcal{N} = 4$ , strong coupling limit known from AdS/CFT correspondence (Maldacena [2]). On the lightcone

$$S_{ws} = \int d\tau \int_0^{p^+} d\sigma \frac{\gamma^2}{2} \left[ \dot{\mathbf{x}}^2 - e^{2\phi} \mathbf{x}'^2 + e^{-\phi} \dot{\phi}^2 - e^{\phi} \phi'^2 \right]$$
$$4\gamma^2 = R^2 T_0 = \sqrt{\alpha_s N_c / \pi} = \sqrt{\lambda}$$

- Strong 't Hooft coupling  $\lambda \gg 1$  is dual to semi-classical limit of string dynamics.
- External  $Q\bar{Q}$  dual to open string with fixed ends on the boundary of AdS<sub>5</sub>.



- Static solution (Rey-Lee, Maldacena [3]):

$$\mathbf{x}' = \mathbf{C}/G^2, \quad \frac{C^2}{G^2} + G\phi'^2 = 2\frac{p^-}{p^+} = \frac{M^2}{p^{+2}}$$

$$E_0(L) = -\frac{(2\pi)^3 \sqrt{\lambda}}{\Gamma(1/4)^4 L}$$

- $E_0$  decreases as  $1/L$  with separation, showing the absence of a confining force.
- Nonetheless, as we shall see, this stretched string has an infinite number of stringy excitations.

- Semi-classical quantization (small oscillations, Callan-Guijosa and Rey-Bak [4]) gives string like modes with discrete levels just above  $E_0$ :

$$E_{N_n} - E_0 = \sum N_n \omega_n$$

$$\omega_n = \frac{(2\pi)^{3/2}}{\Gamma(1/4)^2 L} \xi_n$$

$$\xi_n \sqrt{\xi_n^4 - 1} \int_0^1 \frac{t^2 dt}{[1 + \xi_n^2 t^2] \sqrt{1 - t^4}} = \frac{n\pi}{2}, \quad n = 1, 2, \dots$$

For large  $n$ ,  $\omega_n \sim (2\pi)^3 (n + 1) / \Gamma(1/4)^4 L$ .

Typical of normal modes of a string:

$$T_{eff} \sim 1/L^2.$$

$\mathcal{N} = 4$  is teetering on the brink of quark confinement:

String is there but need a mechanism to prevent  $T_{eff}$  from dropping to 0 when  $L \rightarrow \infty$ .

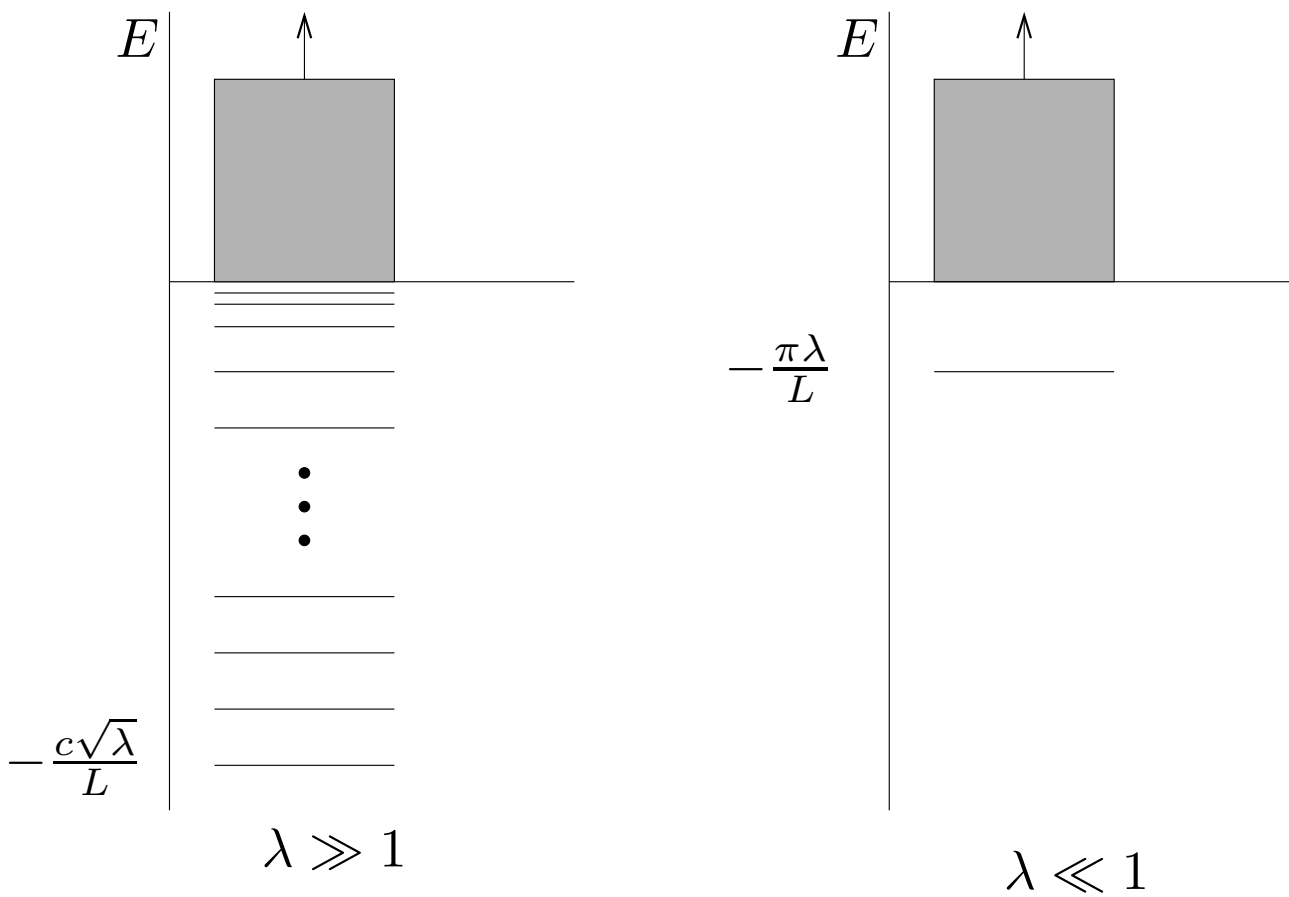
- Near threshold ( $E = 0$ ) discrete levels accumulate (Klebanov, Maldacena, Thorn [6])

$$\frac{E_{n+1}}{E_n} \sim e^{-\pi/2\sqrt{\lambda}}$$

Shown by semiclassical motion of the midpoint of string stretched almost to the horizon ( $\phi = -\infty$ ).

# 8 $\mathcal{N} = 4$ : Strong v.s. Weak

Energy level diagrams for  $N = \infty$



## 9 Discussion

- Transition from weak to strong is simply gradually stronger binding with a deepening gap that eventually supports excited discrete levels that peel off the continuum and move into the gap.
- This nice picture requires  $N = \infty$ . Otherwise the continuum goes all the way down to  $E_0$ .
- Ladder model of Erickson, Seminoff, Szabo, Zarembo [5] captures much of this physics, except at strong coupling there is only a single mode of small oscillations instead of an infinite number of stringy modes  $\omega_n$ .

## 10 Ladder Model

$$\left(-\frac{\partial^2}{\partial t^2} - \frac{\lambda}{L^2 + t^2}\right)\psi(t) = -\frac{E^2}{4}\psi(t) \quad (1)$$

ESSZ show weak coupling result is recovered and ground energy  $\propto -\sqrt{\lambda}/L$  with different numerical coefficient than AdS string.

Can also investigate intermediate coupling (Brower, Tan, Thorn):

- No more bound states for  $\lambda < 1/4$
- For  $\lambda > 1/4$  infinite number of bound states appear with threshold behavior

$$\frac{E_{n+1}}{E_n} \sim e^{-\pi/2\sqrt{\lambda-1/4}}$$

The same strong coupling behavior as  $\mathcal{N} = 4$ .

For intermediate coupling of  $\mathcal{N} = 4$  one has two choices:

- Properly quantize IIB superstring on  $\text{AdS}_5 \times \text{S}_5$  and solve it.
- Figure out how to sum *all* planar diagrams.

Success with 2nd choice for  $\mathcal{N} = 4$  could teach us how to do the same with real QCD at  $N = \infty$ .

The lightcone worldsheet formalism was developed with this second strategy in mind.

(Bardakci, Thorn [7–9])

Implements a gluon chain model of the flux tube  
(Greensite, Thorn [12])

# 11 String Worldsheet

Coordinates and Momenta of String

$$x^\mu(\sigma, t), \quad \mathcal{P}^\mu(\sigma, t)$$

Here  $\mathcal{P}^\mu d\sigma = dp^\mu$  is the energy momentum carried by the element  $d\sigma$  of string.

Nambu-Goto String on the Lightcone:

$$x^+ \equiv \frac{1}{\sqrt{2}}(x^0 + x^3) = t, \quad \mathcal{P}^+ = 1$$

$$S = \int dt \int_0^{p^+} d\sigma \left( \dot{\mathbf{x}} \cdot \mathcal{P} - \frac{1}{2} \mathcal{P}^2 - \frac{T_0^2}{2} \mathbf{x}'^2 \right)$$

$$S \rightarrow \int dt \int_0^{p^+} d\sigma \frac{1}{2} (\dot{\mathbf{x}}^2 - T_0^2 \mathbf{x}'^2)$$

T Duality:  $(\mathbf{q}', \dot{\mathbf{q}}) = (\dot{\mathbf{x}}, -T_0^2 \mathbf{x}')$

$$\begin{aligned} iS &= - \int d\tau \int_0^{p^+} d\sigma \frac{1}{2} (\mathbf{q}'^2 + T_0^{-2} \dot{\mathbf{q}}^2) \\ &\rightarrow - \int d\tau \int_0^{p^+} d\sigma \frac{\mathbf{q}'^2}{2} \quad \text{for } T_0 \rightarrow \infty \end{aligned}$$

# 12 QFT Worldsheet

## Lightcone Propagators

$$\int \frac{dp^-}{2\pi i} \frac{e^{-ix^+ p^-}}{p^2 - i\epsilon} = \frac{\theta(x^+ p^+)}{2|p^+|} \exp \left\{ -ix^+ \frac{\mathbf{p}^2}{2p^+} \right\}$$

$$\int \frac{d\mathbf{p} dp^-}{(2\pi)^3 i} \frac{e^{i\mathbf{x} \cdot \mathbf{p} - ix^+ p^-}}{p^2 - i\epsilon} = \frac{\theta(x^+ p^+)}{4\pi i |x^+|} \exp \left\{ -p^+ \frac{\mathbf{x}^2}{2ix^+} \right\}$$

- Notice the singular behavior as  $x^+ \rightarrow 0$  or  $p^+ \rightarrow 0$ .
- We handle this by discretizing  $p^+ = Mm$  and  $\tau = ix^+ = ka$ , excluding  $M = 0$  and  $k = 0$ .
- We define physical quantities at  $x^+ = 0$  or  $p^+ = 0$  by taking the limit from above *after* the continuum limit.

# 13 QFT Lightcone Worldsheet

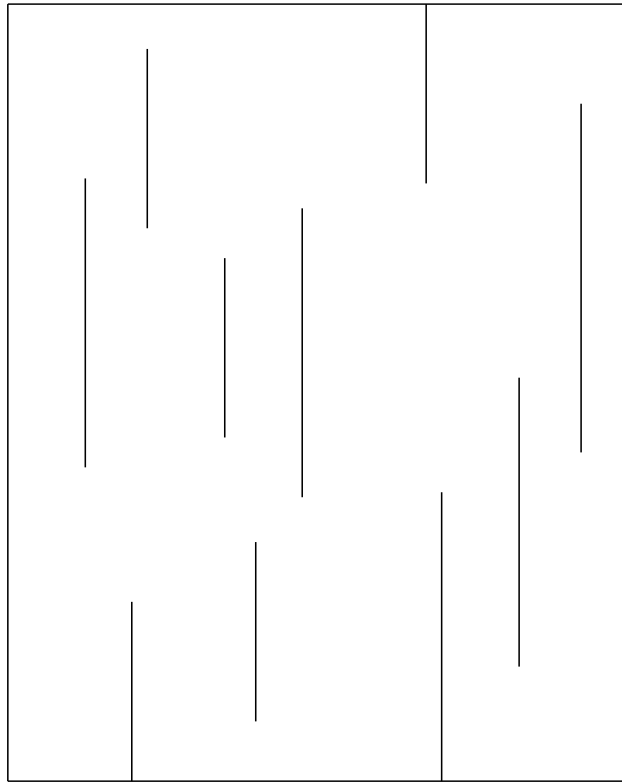
Master Formula for Massless Propagator:

$$\exp \left\{ -\frac{T}{2p^+} \mathbf{p}^2 \right\} = \int_{\substack{\mathbf{q}(0, \tau)=0 \\ \mathbf{q}(p^+, \tau)=\mathbf{p}}} DcDbD\mathbf{q} e^{iS_0}$$

$$iS_0 = \int_0^T d\tau \int_0^{p^+} d\sigma \left( b'c' - \frac{1}{2} \mathbf{q}'^2 \right)$$

- Dirichlet b.c.'s. Cf. string in momentum space
- Represent a field quantum as a composite of String Bits: Total  $p^+ = (\text{Number of bits}) \times m$ .

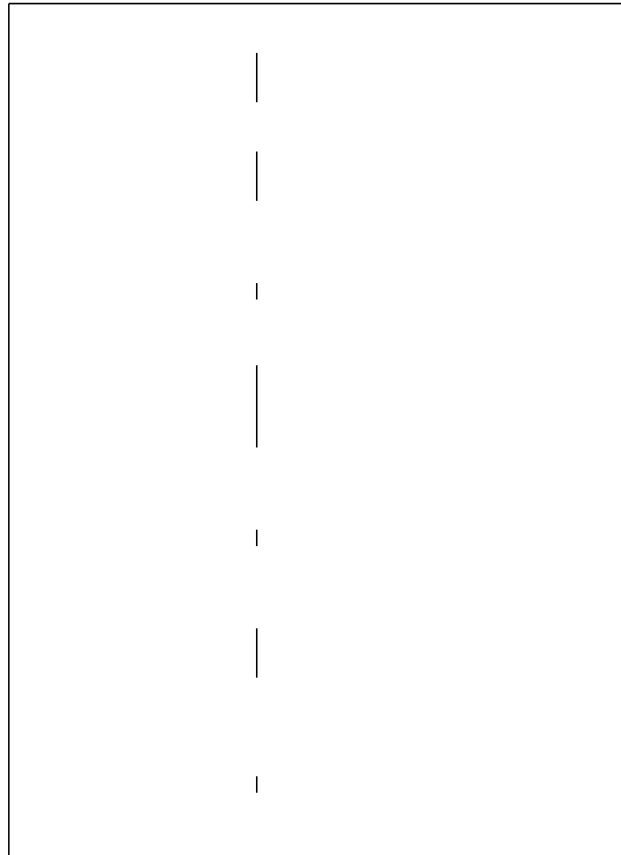
The following is the worldsheet diagram describing a 7 loop 5 gluon diagram:



- LCWS treats:  
 $\mathbf{q}_i$  as worldsheet fields  
 $x^+, p^+$  as moduli of worldsheet
- In string theory one usually first integrates over worldsheet fields, then over moduli.
- Insight on Field/String duality may come from treating loop integrals similarly.

# 14 A mean field approach

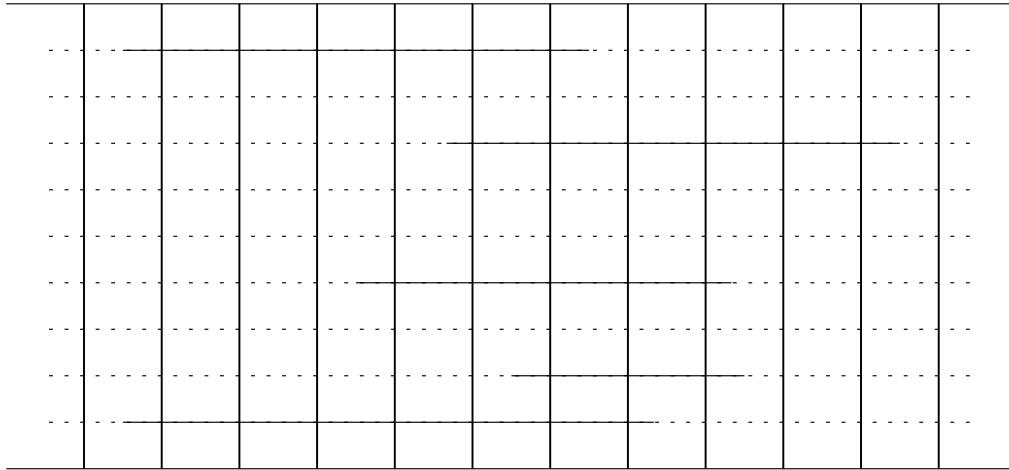
A novel way to resum diagrams (K. Bardakci): First do loop sum at fixed  $\sigma$  in a background mean field:



Then determine mean field self-consistently.  
Note that the diagrams summed here would be tiny bits of self energy bubble diagrams.

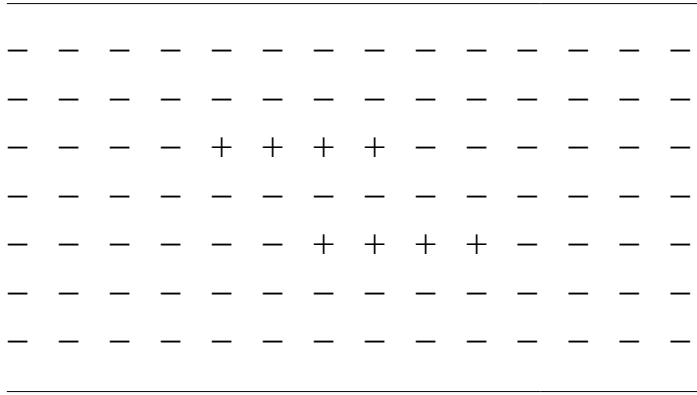
Worksheet Diagram on  $x^+, p^+$  Grid:

$$\sigma = im, \quad \tau = ja$$



- Ising-like spin at each site:  $s_i^j = \pm 1$
- $P_i^j \equiv \frac{1+s_i^j}{2} = 0, 1$
- Internal boundaries (solid lines) correspond to a row of  $+$  spins. Bulk (dotted lines) is a sea of  $-$  spins.

- For example,



is a two loop diagram.

- A factor of coupling  $g$  for each spin flip along a horizontal line.

$$\text{Number of flips} = \sum_{ij} \frac{1 - s_i^j s_i^{j+1}}{2}$$

- Dirichlet b.c.'s on solid lines:

$$\frac{P_i^j P_i^{j-1} a}{2m\epsilon} (\mathbf{q}_i^j - \mathbf{q}_i^{j-1})^2;$$

$\epsilon \rightarrow 0$  forces b.c.'s.

- Can identify an effective dynamical tension

$$\frac{P_i^j P_i^{j-1} a^2}{m^2 \epsilon} \sim T_{eff}^{-2}(s),$$

(Ising spin dependent) string tension.

Cf. AdS radius ( $\phi$ ) dependence of tension  
in AdS/CFT correspondence on light-cone:

$$p^- = \frac{1}{2} \int_0^{p^+} d\sigma [\mathcal{P}^2 + e^{2\phi} \mathbf{x}'^2 + e^\phi (\Pi_\phi^2 + \phi'^2)]$$

# 15 Discussion

- Worldsheet system that sums QFT planar diagrams is a two-dimensional dynamical system of scalar fields  $\mathbf{q}(\sigma, \tau)$ , Grassmann ghosts  $\mathbf{b}(\sigma, \tau)$ ,  $\mathbf{c}(\sigma, \tau)$ , and Ising spins  $s(\sigma, \tau)$ .
- These degrees of freedom have a fairly complicated but *local* worldsheet action.
- The role of “string tension” in this worldsheet system is played by a quantity that depends on the Ising spin configuration. Its “value” fluctuates locally and can’t be regarded as a fixed parameter.
- The fluctuating string tension is the crucial difference between the string description of a field theory and the Nambu-Goto string. We can hopefully trace the well-known short-comings of the Nambu-Goto string for describing hadrons to this difference.

# 16 Conclusion

- Field/String duality promises to shed light on many issues in theoretical physics.
- Most research activity to date has concentrated on the conformal AdS/CFT correspondence, but a worldsheet description can be developed for a much wider class of QFT's.
- As examples, I discussed AdS/CFT applied to the force between static colored sources and the lightcone worldsheet formalism for general field theories.
- I expect field/string duality to yield many more insights in the next few years.

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